

ANOVA: Two-way ANOVA Without Interactions

1. Two-way ANOVA: Example Data and Linear Model

The two-way ANOVA has two categorical independent variables and a quantitative DV. To illustrate the two-way model, consider data in Table 1. The variables are math scores (DV), student sex, and teacher.

Table 1

Math Scores	Student Sex	Teacher	Math Scores	Student Sex	Teacher	Math Scores	Student Sex	Teacher
72	F	Gunther	74	F	Bryan	78	F	Marijke
73	F	Gunther	75	F	Bryan	79	F	Marijke
74	F	Gunther	76	F	Bryan	80	F	Marijke
76	M	Gunther	80	M	Bryan	83	M	Marijke
77	M	Gunther	81	M	Bryan	84	M	Marijke
78	M	Gunther	82	M	Bryan	85	M	Marijke

The correct formatting for both ANOVA and regression can be seen in the SPSS data linked below. Note the structure of the data file – all relevant data are in three columns, one for scores, one for sex, and one for teacher. There are also dummy variables to identify group membership for sex or teacher, but these are for regression and are not needed for ANOVA.

http://www.bwgriffin.com/gsu/courses/edur8132/notes/math_scores.sav

Like regression, the ANOVA model can be displayed symbolically in linear equation form, as shown below.

$$Y_{ijk} = \mu + \alpha_j + \beta_k + \varepsilon_{ijk}$$

where

Y_{ijk} = is the math score for student i with teacher j with sex k ,

μ = grand mean across all classes and sexes in the sample,

α_j = the mean difference from μ , or effect, for instructor j ,

β_k = the mean difference from μ for sex k , and

ε_{ijk} = is the error term, or how far each math score deviates from $(\mu + \alpha_j + \beta_k)$ (Glass & Hopkins, 1984)

Factors and Blocking Variables

In ANOVA parlance categorical predictor variables are called **factors** if they are of primary interest in the research and a **blocking variable** if included because they are a source of variation in the DV that must be controlled. In essence, a blocking variable serves the same purpose as a **covariate** in ANCOVA, i.e., it is an important variable to include in the model but is not of primary interest for the research. By including the blocking variable, the model reduces a source of variation that could bring clarity to the analysis by providing more power (probability of rejecting a false null) and precision (less model error and tighter confidence intervals).

In this presentation all categorical predictors will be called factors.

Note: The data for this example were first presented in the regression analysis with two categorical variables. Results of that regression analysis will be compared to ANOVA results to show analysis similarities. Those interested can directly access the presentation and videos in the links below.

Document: [Regression with Multiple Categorical Predictors](#) (PDF)

- Video: [Two Categorical Variables Part 1](#) (27 minutes)
- Video: [Two Categorical Variables Part 2](#) (11 minutes)

2. Two-way ANOVA Hypotheses

The hypotheses remain essentially unchanged from one-way ANOVA except comparisons denote taking means across categories, or cells, of the second factor.

(a) Test for Factor α : This test compares means across J levels of first factor (J row population means)

$$H_0: \mu_{1.} = \mu_{2.} = \dots = \mu_{j.}$$

H_a : not all means of J levels are equal

Where the period subscript, ., represents averaging means across all levels of the second factor.

For the current example the null for teacher is

$$H_0: \mu_{1.} = \mu_{2.} = \mu_{3.} \text{ (taking the mean across both males and females for each teacher)}$$

H_a : not all of the teachers have equal mean math scores.

(b) Test for Factor β : This test compares means across K levels of the second factor (K column population means)

$$H_0: \mu_{.1} = \mu_{.2} = \dots = \mu_{.k}$$

H_a : not all means of K levels are equal

For the current example the null for the second factor, student sex, is:

$$H_0: \mu_{1.} = \mu_{2.} \text{ (taking the mean across all instructors)}$$

$H_a: \mu_{1.} \neq \mu_{2.}$

(c) Test the Interaction Between the two Factors

By default, and consistent with tradition, most ANOVA software automatically tests the interaction between the two factors. Briefly explained, an interaction occurs for two factors when the mean difference between levels of one factor varies across levels of the second factor.

To illustrate, assume that the marginal mean difference in math scores between males and females is 5 points. If that 5-point mean difference occurs for each instructor, then there is no interaction because the sex difference is constant for each instructor. Graphically that constant mean difference is illustrated in Figure 1. Note that the plotted lines showing female and male performance are parallel – the gap between female and male remains the same.

If, however, the difference between female and male students varies across instructors (e.g., 1 point difference in Gunther's class, but an 8-point difference between females and males in Bryan's class), then an interaction occurs between the two factors. Such an interaction is illustrated in Figure 2. Note that the plotted lines showing sex differences are no longer parallel – the gap between female and male students differs across instructors.

For the remainder of this presentation no interaction will be assumed to simplify discussion of the two-way ANOVA. Interactions will be covered in detail in other presentations.

Figure 1: No Interaction between Student Sex and Instructor

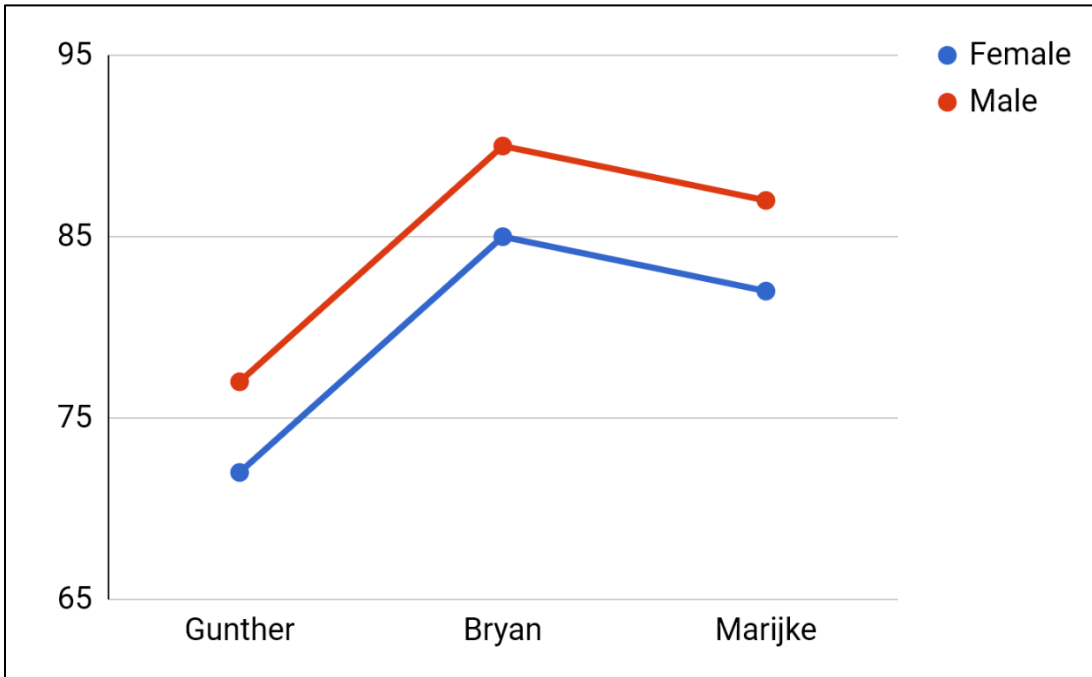
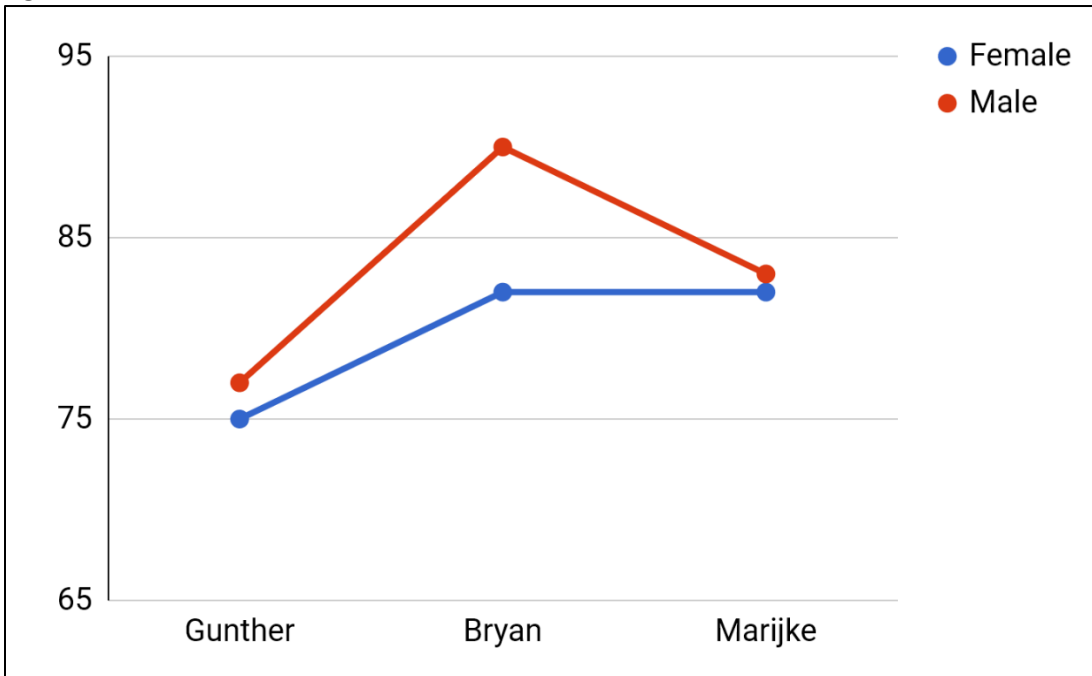


Figure 2: Interaction between Student Sex and Instructor



3. ANOVA Computation

As before, ANOVA computation is based upon the information found in the summary table below. This table shows sums of squares (SS), degrees of freedom (df), mean squares (MS), and F-ratios (F).

Table 2: Two-way ANOVA Summary Table

Source	SS	df	MS	F
Factor A	SS_A	$df_A = j - 1$	SS_A/df_A	MS_A/MS_w
Factor B	SS_B	$df_B = k - 1$	SS_B/df_B	MS_B/MS_w
Interaction A×B	SS_{AB}	$df_{AB} = (j - 1)(k - 1)$	SS_{AB}/df_{AB}	MS_{AB}/MS_w
Within (error, residual)	SS_w	$df_w = jk(n - 1)$	SS_w/df_w	
total	SS_T	$df_t = n - 1$		

However, for this presentation the model is simplified to include only the main effects of each factor, so the interaction is removed. The resulting ANOVA summary table appears in Table 3.

Table 3: Two-way ANOVA Summary Table without an Interaction

Source	SS	df	MS	F
Factor A	SS_A	$df_A = j - 1$	SS_A/df_A	MS_A/MS_w
Factor B	SS_B	$df_B = k - 1$	SS_B/df_B	MS_B/MS_w
Within (error, residual)	SS_w	$df_w = jk(n - 1)$	SS_w/df_w	
total	SS_T	$df_t = n - 1$		

4. Software Commands

Below are results from both SPSS and JASP. Screenshots of software commands and results are shown first, then explanations of results are provided in sections that follow.

For the two-way ANOVA model the following information is needed from software:

- ANOVA summary table with SS, df, MS, F-ratios, and p-values;
- descriptive statistics;
- effect sizes (ΔR^2 and η^2 , and possibly η_p^2);
- marginal means; and
- multiple pairwise comparisons tables.

As with regression, one should also check model assumptions. That is covered in detail in a separate presentation.

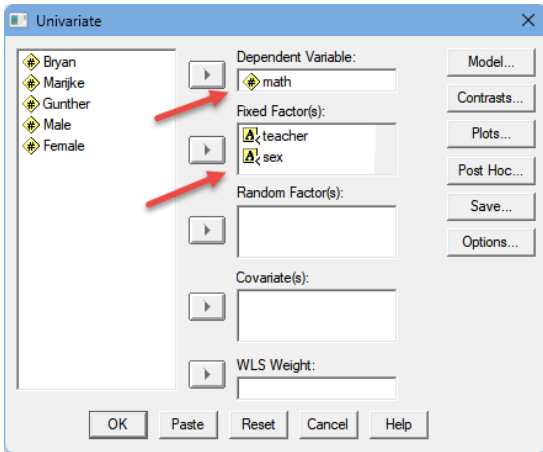
Note: Show both SPSS and JASP walkthroughs for ANOVA

SPSS

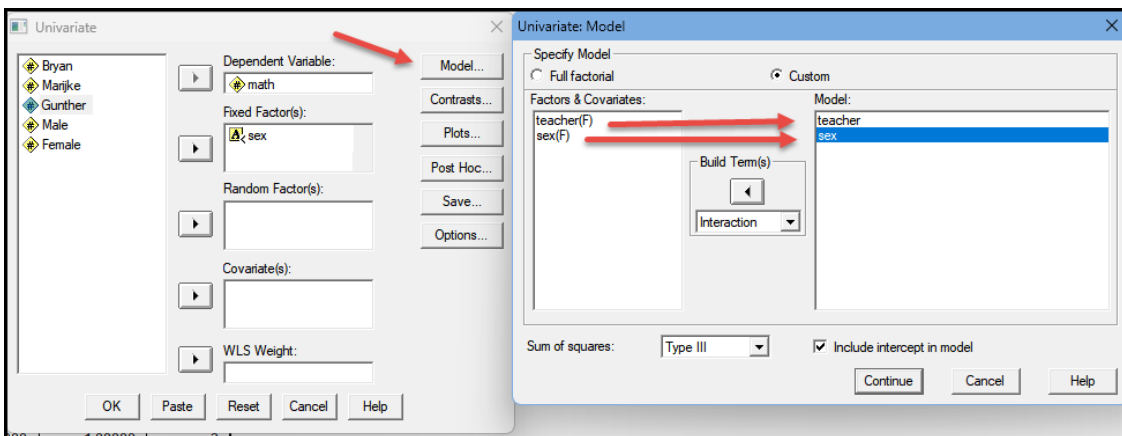
Below are commands and screenshots showing how to obtain two-way ANOVA results.

Analyze → General Linear Mode → Univariate

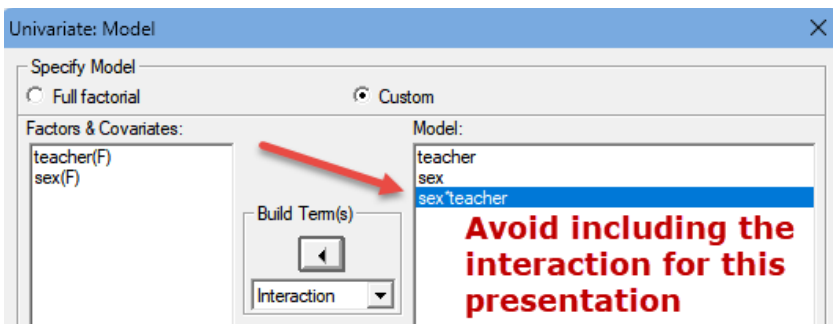
Then move the DV (math) to the **Dependent Variable** box, and the factors (teacher, sex) **Fixed Factors** box.



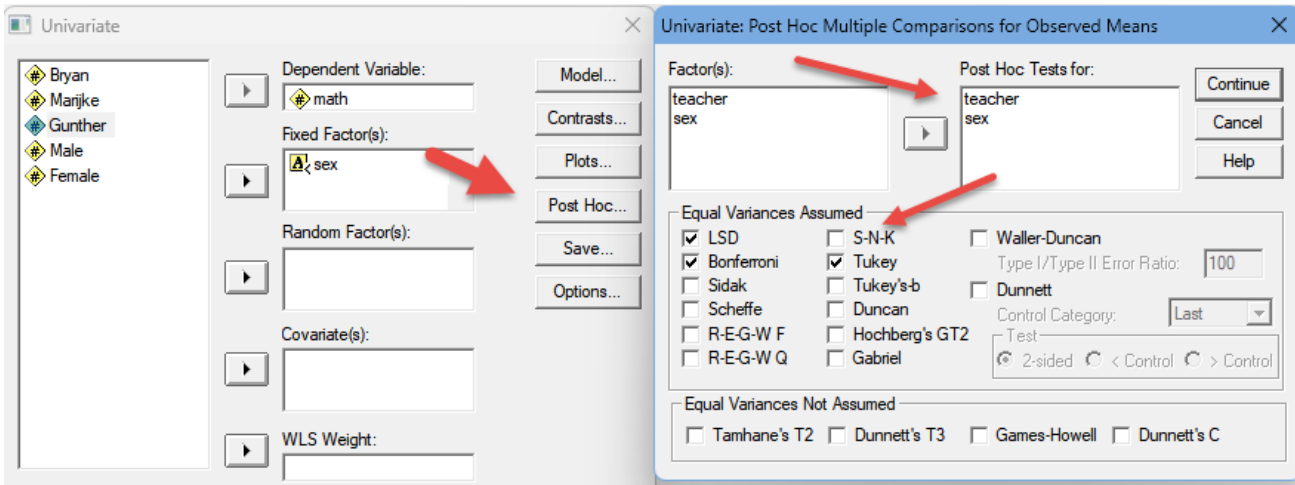
Next, select **Model**, then select **Custom**, then move each factor, **teacher** and **sex**, to the **Model** box. This avoids producing the default interaction. Complete this step by clicking on **Continue**.



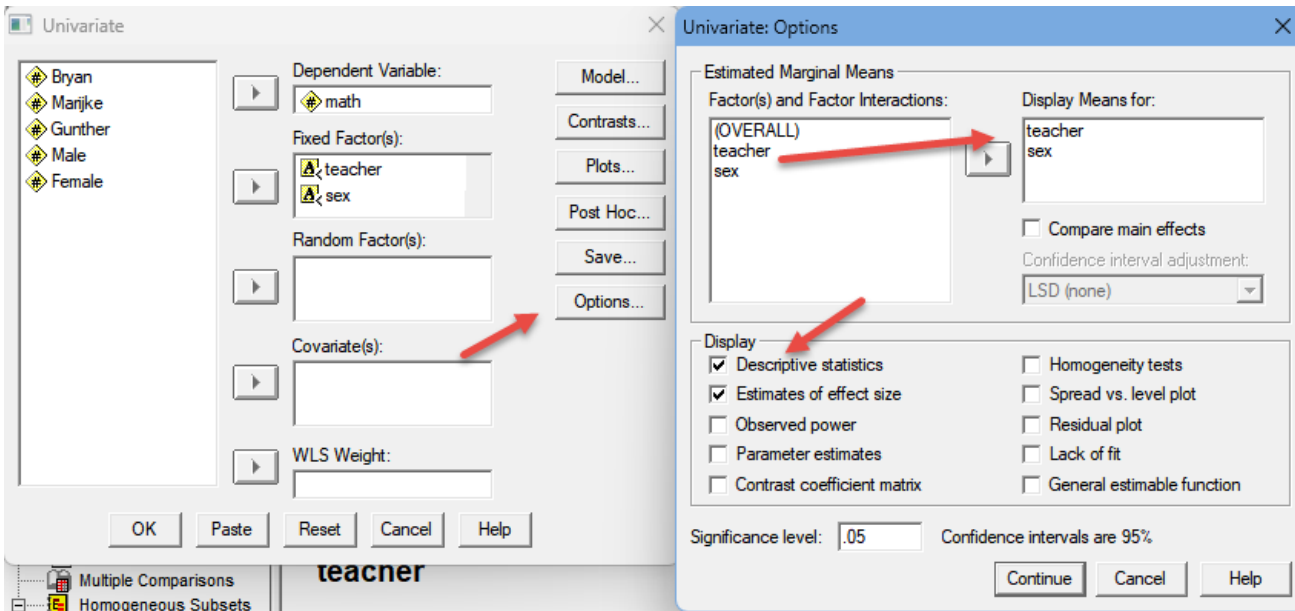
If both **teacher** and **sex** were highlighted together and moved to the **Model** box, it would create the interaction between the two factors and would look like the screenshot below. At this point do not include the interaction because that is discussed in a separate presentation.



The next step is to select **Post Hoc**, then move **Teacher** to the **Post Hoc Tests** box. Next, select whichever multiple comparison procedure is desired: **Bonferroni**, **Tukey**, **Scheffé**, etc. This will provide a table of multiple comparisons with Type 1 error rates adjusted for the number of comparisons. **Sex** can also be moved to the **Post Hoc Tests** box, but since it has only two categories multiple comparisons are not needed, and SPSS likely won't provide any output for **sex**.



Next, select **Options**, then move **Sex** and **Teacher** to **Display Means** box, and select **Descriptive Statistics**, **Estimates of Effect Size**, and anything you wish. Then select **Continue**.



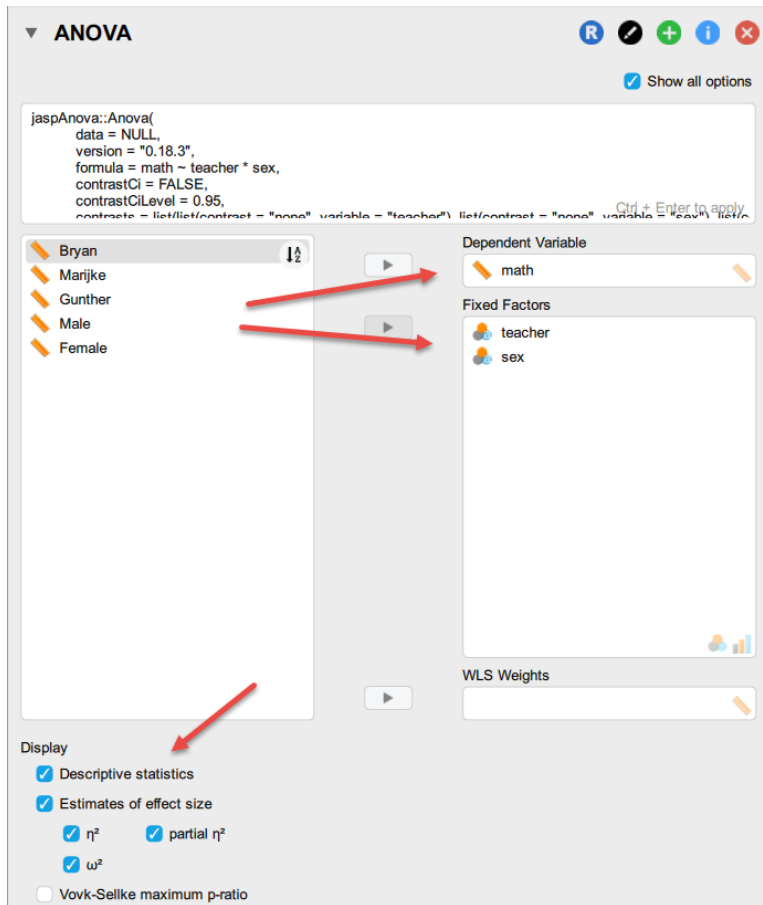
To complete the analysis, select **OK** to run the ANOVA command.

JASP

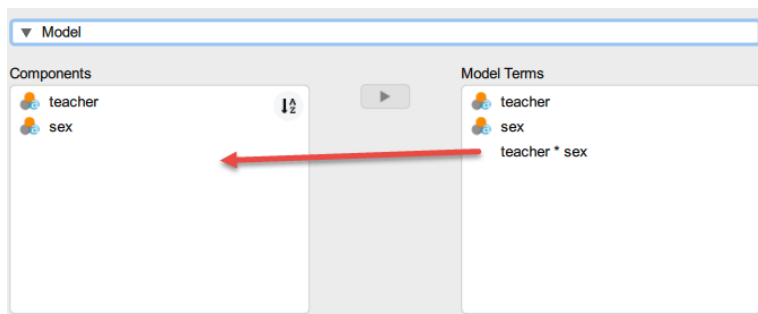
Below are commands and screenshots showing how to obtain two-way ANOVA results in JASP.

ANOVA → ANOVA (under Classic)

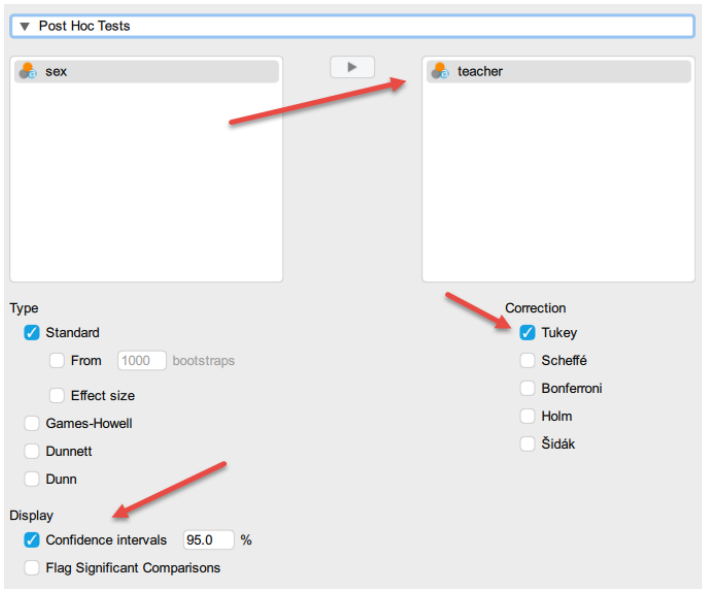
Then move the DV (**math**) to the **Dependent Variable** box, and the factors (**teacher, sex**) **Fixed Factors** box. Next, select **Descriptive Statistics** and each of the **Effect Size** options.



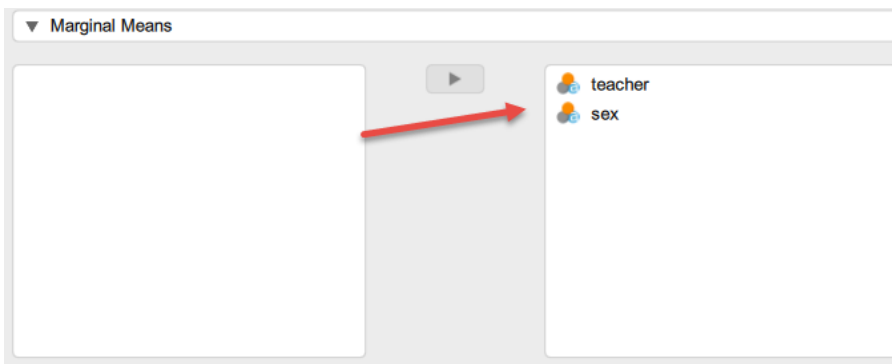
Next, select **Model**, then remove the interaction term from the **Model Terms** box (i.e., double click or drag “**teacher*sex**” interaction to remove it from the **Model Terms** box). Once completed, the **Model** command should have only **teacher** and **sex** in the **Model Terms** box.



Next, select **Post Hoc Tests**, then move **teacher** to the right box, select **Tukey** as the **Correction** type (since JASP does not provide confidence intervals for Scheffé or Bonferroni), then select **Confidence Intervals**.



Last, select **Marginal Means** to get overall means on the DV for each factor and category. Move both **teacher** and **sex** to the right box.



5. Statistical Inference: Overall Model Fit

Like with regression one may perform statistical inferences on the overall model (although this is less a focus in ANOVA literature), predictors, and group comparisons. In this section overall model fit is examined.

Model Fit Statistics: R^2 and η^2 , and Adj. R^2 and ε^2

With ANOVA one will see reference to η^2 (eta squared) which is analogous to R^2 . The overall model test may be formulated in terms of η^2 rather than R^2 ,

$$H_0: \eta^2 = 0.00$$

which has the same interpretation as the R^2 null (i.e., $H_0: R^2 = 0.00$, none of the model factors are related to the dependent variable).

The formula for R^2 and η^2 is the ratio Corrected Model SS / Corrected Total SS,

$$R^2 = \eta^2 = \frac{\text{corrected model SS}}{\text{corrected total SS}}.$$

Adjusted R^2 also goes by a different name in ANOVA and is called epsilon squared, ε^2 . Two values are needed to calculate adj. R^2 , the MSE and the variance of the dependent variable. The formula for adj. R^2 and ε^2 is

$$\text{adj. } R^2 = \varepsilon^2 = 1 - \frac{MSE}{\text{Var}(DV)}$$

where MSE is the mean squared error (or MS residuals, or MS within) from the ANOVA table, and $\text{Var}(DV)$ is the variance of the DV.

SPSS ANOVA

SPSS provides both R^2 and adjusted R^2 , and the F test for the model fit. Look for the line called Corrected Model in the SPSS ANOVA summary table below. The F ratio for the model is 74.511 which is significant at the .01 level. This indicates that at least one of the factors, teachers or student sex, demonstrates mean difference in math scores.

Dependent Variable: math						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	239.500 ^a	3	79.833	74.511	.000	.941
Intercept	109980.500	1	109980.500	102648.5	.000	1.000
teacher	127.000	2	63.500	59.267	.000	.894
sex	112.500	1	112.500	105.000	.000	.882
Error	15.000	14	1.071			
Total	110235.000	18				
Corrected Total	254.500	17				

a. R Squared = .941 (Adjusted R Squared = .928)

Using the formulas both R^2 and adjusted R^2 can be calculated. The value for R^2 is

$$R^2 = \eta^2 = \frac{\text{corrected model SS}}{\text{corrected total SS}} = \frac{239.5}{254.5} = .941.$$

which agrees with the R^2 in the SPSS output above.

The SD for math scores is 3.86918, and the variance is the SD^2 , so the variance of math scores is $3.86918^2 = 14.9705$. The ANOVA summary table provides the MSE which is 1.071. The adj R^2 is calculated below.

$$\text{adj. } R^2 = \varepsilon^2 = 1 - \frac{MSE}{\text{Var}(DV)} = 1 - \frac{1.071}{14.9705} = .9284$$

JASP ANOVA

The JASP ANOVA summary table, below, does not provide an assessment of the overall model fit: there is no F ratio, η^2 , or ε^2 .

Cases	Sum of Squares	df	Mean Square	F	p	η^2	η^2_p	ω^2
teacher	127.000000000	2	63.500000000	59.266666667	< .001	0.499017682	0.894366197	0.488541084
sex	112.500000000	1	112.500000000	105.000000000	< .001	0.442043222	0.882352941	0.435997764
Residuals	15.000000000	14	1.071428571					

Note. Type III Sum of Squares

It is possible, however, to calculate each of these from the information provided and the SD or variance of the DV.

Adjusted R^2 and ε^2

This value is the easier to calculate since it requires only the addition of the SD or variance of the DV. The calculation was illustrated above. The MSE in JASP is in the line labeled residuals (i.e., recall that this line can be called within, error, or residual), and is the same value, 1.071, as reported by SPSS.

R² and η^2

To calculate these values, one must first calculate the total SS and the corrected model SS. To find these, the following values are needed:

- variance of the dependent variable, var(DV);
- total sample size minus one, $n - 1$; and
- error SS, also called residual SS and within SS.

Values for the current data:

- variance = SD^2 ; SD for math scores is 3.86918, so $SD^2 = 3.86918^2 = 14.9705$;
- total sample size is 18, so $n - 1 = 17$; and
- error SS is provided by JASP and called the residual SS = 15.00.

With these values it is now possible to calculate both the corrected total SS and the corrected model SS.

- Corrected Total SS = $\text{var}(DV) * n-1 = 14.9705 * 17 = 254.4985$
- Corrected Model SS = Corrected Total SS – Error SS = $254.4985 - 15.00 = 239.4985$

Plugging these values into the equation below produces the $R^2 = \eta^2$.

$$R^2 = \eta^2 = \frac{\text{corrected model SS}}{\text{corrected total SS}} = \frac{239.4985}{254.4985} = .941$$

Model F Test

Lastly, for those interested, it is also possible to perform the model F-test using the information calculated above and in the JASP ANOVA summary.

Values needed:

- Model SS which calculated above for the R^2 formula as 239.4985;
- Model df which is the sum of factors (and covariates, if present) df which is $2 + 1 = 3$; and
- error (residual) mean squared error which is given by JASP and is 1.0714.

With this information it is possible to calculate the F ratio for the model SS.

First, calculate the mean square for the model which is the model SS / model df, i.e.,

$$\text{Mean Square Model} = \text{Model SS} / \text{Model df} = 239.4985 / 3 = 79.8328$$

which agrees with SPSS's results. Next, calculate the F ratio for the model which is the model MS / MSE, i.e.,

$$F = \text{model MS} / \text{MSE} = 79.8328 / 1.0714 = 74.512$$

which also agrees with SPSS. One would then compare this calculated F ratio to the critical F value with 3 and 14 df to determine statistical significance.

Faster Option

Of course, it is also possible to obtain the model F test and fit statistics by using regression. Each value calculated above is reported by the JASP regression summary below.

Model Summary - math					
Model	R	R ²	Adjusted R ²	RMSE	
H ₀	0.000000000	0.000000000	0.000000000	3.869184441	
H ₁	0.970082937	0.941060904	0.928431097	1.035098339	

ANOVA						
Model		Sum of Squares	df	Mean Square	F	p
H ₁	Regression	239.500000000	3	79.833333333	74.511111111	< .001
	Residual	15.000000000	14	1.071428571		
	Total	254.500000000	17			

Note. The intercept model is omitted, as no meaningful information can be shown.

Coefficients						
Model		Unstandardized	Standard Error	Standardized ^a	t	p
H ₀	(Intercept)	78.166666667	0.911975519		85.711365139	< .001
H ₁	(Intercept)	75.500000000	0.487950036		154.728956566	< .001
	sex (M)	5.000000000	0.487950036		10.246950766	< .001
	teacher (Gunther)	-3.000000000	0.597614305		-5.019960159	< .001
	teacher (Marijke)	3.500000000	0.597614305		5.856620186	< .001

^a Standardized coefficients can only be computed for continuous predictors.

6. Statistical Inference: Factors and Covariates

For most researchers the primary interest lies in tests for model factors and covariates. For factors in ANOVA the null states that group means are the same. For the teacher factor, they indicate each teacher has the same marginal mean in math scores, i.e.,

$$H_0: \mu_1 = \mu_2 = \mu_3.$$

where the period subscript indicates scores are averaged over males and females. For the sex factor, the null states each sex's marginal mean is the same, i.e.,

$$H_0: \mu_1 = \mu_2.$$

where these means are averaged, separately for females and males, across teacher groups. The word marginal in ANOVA indicates means are taken across groups in other factors. For example, instead of examining the mean math score for males in Bryan's class, and in Gunther's class, and in Marijke's class, the mean for males is taken across all three classes to provide an overall mean for males. Then the same type of overall mean is calculated for females, and it is these two overall means, or marginal means, that are compared between the sexes.

Inferential tests in ANOVA are performed with partial F tests. Both JASP and SPSS summary tables are reported below. Both variables, teacher and student sex, are statistically significant: teacher ($F = 59.266$, $p < .001$) and sex ($F = 105.00$, $p < .001$). These results indicate group means differ by teacher and by sex.

JASP

Cases	Sum of Squares	df	Mean Square	F	p	η^2	η_p^2	ω^2
teacher	127.000000000	2	63.500000000	59.266666667	< .001	0.499017682	0.894366197	0.488541084
sex	112.500000000	1	112.500000000	105.000000000	< .001	0.442043222	0.882352941	0.435997764
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Note. Type III Sum of Squares

SPSS

Dependent Variable: math						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
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Intercept	109980.500	1	109980.500	102648.5	.000	1.000
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sex	112.500	1	112.500	105.000	.000	.882
Error	15.000	14	1.071			
Total	110235.000	18				
Corrected Total	254.500	17				

a. R Squared = .941 (Adjusted R Squared = .928)

7. Effect Sizes

Many fields of research expect statistical analyses to present effect size measures. For reporting global effects of a factor, three measures are common with ANOVA:

- eta squared, η^2 ,
- partial eta squared, η_p^2 , and
- omega squared, ω^2 .

Two of these will be briefly described here, η^2 and η_p^2 . For those interested in ω^2 , which is a slightly reduced form of η_p^2 , see Maxwell, et al. (1981) and Kroes and Finley (2023). JASP provides all three measures and SPSS (version 12, my version) provides only η_p^2 . Perhaps newer versions of SPSS offer other effect size options.

For the overall model η^2 , it is the same as R^2 , so it represents the proportion of variance in the DV that is accounted for by the factors and covariates in the ANOVA mode.

When η^2 is calculated for a factor, or covariate, it represents the same information a ΔR^2 , the proportion of variance predicted in the DV that is uniquely attributed to this factor or covariate after controlling for other model factors and covariates. The symbol can be confusing since η^2 does not use the change symbol, Δ .

Example:

- $\Delta R^2 = .499$ for teacher in regression model of math scores by student sex and teacher
- $\eta^2 = .499$ for teacher in the ANOVA model of math scores by student sex and teacher
- see JASP ANOVA output above for η^2 values

The partial eta squared, η_p^2 , is the squared partial correlation between the DV and a factor or covariate controlling for other variables in the ANOVA or regression equation. For example, the correlation between sex (using the male dummy variable) and math scores is .6648, but when teacher is partial from the correlation the partial r increases to .93933 (see JASP output below for partial correlation command).

The square of the partial r = $.93933^2 = .8823$, which is the same value reported for sex by JASP and SPSS in the ANOVA summary output above.

The screenshot shows the JASP software interface. On the left, the 'Correlation' dialog box is open, showing the following configuration:

- Code: `jaspRegression::Correlation(data = NULL, version = "0.18.3", alternative = "twoSided", assumptionCheckMultivariateEnergy = FALSE, assumptionCheckMultivariateMardia = FALSE, assumptionCheckMultivariatePovstov = FALSE)`
- Variables: math, Male
- Partial out: Bryan, Marijke

On the right, the output window displays the following information:

Standardized coefficients can only be computed for continuous

Correlation

```
jaspRegression::Correlation(
  version = "0.18.3",
  partialOutVariables = list("Bryan", "Marijke"),
  variables = list("math", "Male"))
```

Pearson's Partial Correlations

Variable		math	Male
1. math	Pearson's r	—	—
	p-value	—	—
2. Male	Pearson's r	0.939336437	—
	p-value	< .001	—

Note. Conditioned on variables: Bryan, Marijke.

Calculation of η^2 for a factor or covariate is simply the SS of that effect divided by the total SS. Calculation of the total SS was presented above and is 254.50. Calculation of η^2 is shown below for sex and teacher.

- Sex $\eta^2 = \text{sex ss} / \text{total ss} = 112.50 / 254.50 = .4429$
- Teacher $\eta^2 = \text{teacher ss} / \text{total ss} = 127.50 / 254.50 = .5009$

8. Statistical Inference: Multiple Pairwise Comparisons

If a factor with more than two groups is significant, one should explore group differences using a multiple comparisons procedure that controls for inflation of the Type 1 error rate. Three procedures, the Bonferroni, Scheffé, and Tukey honest significant difference (HSD), have been reviewed in previous presentations and won't be elaborated upon here.

While both sex and teacher are significant, only teacher requires a multiple comparison procedure to protect against inflation of the Type 1 error rate since it has more than one pairwise comparison. For each pairwise comparison the null specifies that the two means are the same:

$$H_0: \mu_j = \mu_k \text{ (where } j \text{ and } k \text{ refer to different groups).}$$

Note: Illustrate obtaining multiple pairwise comparisons with JASP and SPSS.

Pairwise Comparisons

Multiple pairwise comparisons with Type 1 inflation control procedures are shown below from SPSS and JASP. SPSS provides confidence intervals for all three of the control procedures while JASP provides confidence intervals only for Tukey's approach.

JASP Pairwise Comparison

Post Hoc Comparisons - teacher							
		Mean Difference	95% CI for Mean Difference		SE	t	P _{tukey}
			Lower	Upper			
Bryan	Gunther	3.000000000	1.435875737	4.564124263	0.597614305	5.019960159	< .001
	Marijke	-3.500000000	-5.064124263	-1.935875737	0.597614305	-5.856620186	< .001
Gunther	Marijke	-6.500000000	-8.064124263	-4.935875737	0.597614305	-10.876580345	< .001

Note. P-value and confidence intervals adjusted for comparing a family of 3 estimates (confidence intervals corrected using the tukey method).

Note. Results are averaged over the levels of: sex

SPSS Pairwise Comparisons

Dependent Variable: math							
	(I) teacher	(J) teacher	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	Bryan	Gunther	3.0000*	.59761	.001	1.4359	4.5641
		Marijke	-3.5000*	.59761	.000	-5.0641	-1.9359
	Gunther	Bryan	-3.0000*	.59761	.001	-4.5641	-1.4359
		Marijke	-6.5000*	.59761	.000	-8.0641	-4.9359
	Marijke	Bryan	3.5000*	.59761	.000	1.9359	5.0641
		Gunther	6.5000*	.59761	.000	4.9359	8.0641
Scheffe	Bryan	Gunther	3.0000*	.59761	.001	1.3658	4.6342
		Marijke	-3.5000*	.59761	.000	-5.1342	-1.8658
	Gunther	Bryan	-3.0000*	.59761	.001	-4.6342	-1.3658
		Marijke	-6.5000*	.59761	.000	-8.1342	-4.8658
	Marijke	Bryan	3.5000*	.59761	.000	1.8658	5.1342
		Gunther	6.5000*	.59761	.000	4.8658	8.1342
Bonferroni	Bryan	Gunther	3.0000*	.59761	.001	1.3758	4.6242
		Marijke	-3.5000*	.59761	.000	-5.1242	-1.8758
	Gunther	Bryan	-3.0000*	.59761	.001	-4.6242	-1.3758
		Marijke	-6.5000*	.59761	.000	-8.1242	-4.8758
	Marijke	Bryan	3.5000*	.59761	.000	1.8758	5.1242
		Gunther	6.5000*	.59761	.000	4.8758	8.1242

Based on observed means.

*. The mean difference is significant at the .05 level.

Results show that for every comparison there is a significant difference. In such a situation it can be helpful to provide the overall marginal means for each group so readers can easily understand how each group performed. Marginal means are provided by both JASP and SPSS. JASP's marginal means are posted below. Note that marginal means in ANOVA typically equal to observed means unless there are unequal sample size across cells and/or covariates present. A cell is a combination of factor categories such as females in Bryan's class, females in Gunther's class, females in Marijke's class, and then repeat for each combination of male students and instructors. For a 3x2 ANOVA (3 categories in the first factor and 2 in the second factor) there are 3x2 = 6 cells.

Marginal Means - teacher				
teacher	Marginal Mean	95% CI for Mean Difference		SE
		Lower	Upper	
Bryan	78.000000000	77.093662203	78.906337797	0.422577127
Gunther	75.000000000	74.093662203	75.906337797	0.422577127
Marijke	81.500000000	80.593662203	82.406337797	0.422577127

9. Reporting in APA Style

Each of the primary components of ANOVA are explained above. It is now possible to bring each together to report results in APA style.

Note: Illustrate in both JASP and SPSS how to obtain the following:

- ANOVA summary table,
- test for interaction and remove if insignificant,
- group descriptive statistics,
- marginal means,
- model fit information (calculate R^2 and adj. R^2 for JASP), and
- multiple comparison table.

Table 4: ANOVA Results and Descriptive Statistics for Math Scores by Instructor and Student Sex

Factors	Mean	Adjusted Mean	SD	n	
Student Sex					
Female	75.666	see	2.783	9	
Male	80.666	note	3.162	9	
Instructor		below			
Bryan	78.00		3.405	6	
Gunther	75.00		2.366	6	
Marijke	81.50		2.880	6	
Source	SS	df	MS	F	η^2
Instructor	127.000	2	63.500	59.266*	.499
Student Sex	112.500	1	112.500	105.000*	.442
Error	15.000	14			

Note: $\eta^2 = .941$, $\varepsilon^2 = .928$. Two-way interaction tested and not significant: $F = 1.50$, $p = .262$.

* $p < .05$

(Note: The **adjusted mean column** is only needed if the marginal means differ from the observed means. As noted above, marginal means typically differ from observed means only when there are unequal cells sizes or unequal group sizes, or if a covariate is present. If there is no difference between marginal and adjusted means it is best to eliminate the adjusted mean column.)

Table 5: Comparisons of Mean Differences in Math Scores by Instructor

Instructor Comparison	Estimated Mean Difference	Standard Error of Difference	Tukey Adjusted 95% CI
Bryan vs. Gunther	3.000	0.597	1.435, 4.564
Bryan vs. Marijke	-3.500	0.597	-5.064, -1.935
Gunther vs. Marijke	-6.500	0.597	-8.064, -4.935

Note: Bryan = instructor Bryan, Gunther = instructor Gunther, and Marijke = instructor Marijke.

* $p < .05$, where p-values are adjusted using the Tukey HSD method.

ANOVA results show that math scores differ by both student sex and instructor. Males demonstrated greater achievement ($M = 80.66$, $SD = 3.16$) than females ($M = 75.66$, $SD = 2.78$). Each pairwise comparison among instructors is statistically significant. Among instructors, students in Marijke's class demonstrated the highest math scores, students in Bryan's class the next highest, and students in Gunther's class performed lowest.

10. Comparison Between ANOVA and Regression.

As previously noted, the data in this example were used to illustrate regression with two categorical variables. Below are screenshots of regression results and the link to the presentation.

[Regression with Multiple Categorical Predictors](#) (PDF)

Variable	b	se	ΔR^2	95% CI	F	t
Male	5.00	0.49	.44	3.95, 6.05		10.25*
Teacher			.50		59.27*	
Bryan	3.00	0.60		1.72, 4.28		5.02*
Marijke	6.50	0.60		5.22, 7.78		10.88*
Intercept	72.50	0.49		71.45, 73.55		148.58*

Note: $R^2 = .94$, adj. $R^2 = .93$, $F_{3,14} = 74.51^*$, $MSE = 1.071$, $n = 18$. ΔR^2 represents the semi-partial correlation or the increment in R^2 due to adding the respective variable. Male (male = 1, female = 0), Bryan (=1, others = 0) and Marijke (=1, others = 0) are dummy variables.
* $p < .05$.

Comparison	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
Bryan vs. Gunther	3.00*	0.598	1.38, 4.62
Marijke vs. Gunther	6.50*	0.598	4.88, 8.12
Bryan vs. Marijke	-3.50*	0.598	-5.12, -1.88

* $p < .05$, where p-values are adjusted using the Bonferroni method.

Regression results show that both student sex and teachers are statistically related to students' math scores at the .05 level of significance. Males score about 5 points higher than females, and students in Marijke's class tend to score higher than students in either of Bryan's or Gunther's class. Students in Gunther's class score lower than in either Bryan's or Marijke's class. Note that all teacher comparisons are statistically different.

ANOVA vs Regression Comparisons

- Model fit: same for both regression and ANOVA with $R^2 = .94$ and adjusted $R^2 = .93$
- Inferential Test: Sex $t = 10.25$ is same as ANOVA F test 105.0 when squared ($t^2 = 10.25^2 = 105$)
- Inferential Test: Instruction, same F ratio, 59.27
- Inferential Test: Overall model, same F ratio, 74.51 (see SPSS output for model test)
- Effect Sizes: ΔR^2 for sex and instructor same as η^2 , .50 and .44 respectively
- MSE same for both: 1.071
- Pairwise Comparisons: same except for order of comparison (e.g., M vs G mean difference = 6.5 vs G vs. M mean difference = -6.5, and confidence intervals same expect for sign of limits and difference due to correction procedure, Bonferroni vs Tukey)

As these comparison demonstrate, ANOVA and regression are mathematically the same.

References

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