

## **$\Delta R^2$ Explained and Illustrated**

### **1. $\Delta R^2$ Defined**

**Recall that  $R^2$  is a measure of the proportion of variability the DV that is predicted by the model IVs.**

**$\Delta R^2$  is the change in  $R^2$  values from one model to another.**

**$\Delta R^2$  is the incremental increase in the model  $R^2$  resulting from the addition of a predictor, or set of predictors, to the regression equation.**

## 2. Example

### Model 1 (Reduced model)

$$\text{Test Scores} = b_0 + b_1 (\text{IQ}) + e$$

**DV = Student Reading Test Scores**

**IV 1 = IQ**

### Model 2 (Full model)

$$\text{Test Scores} = b_0 + b_1 (\text{IQ}) + b_2 (\text{Study Time}) + e$$

**DV = Student Reading Test Scores**

**IV 1 = IQ**

**IV 2 = Amount of time spent studying before test**

<b>Models</b>	<b>R<sup>2</sup></b>
<b>Full: Test Scores = <math>b_0 + b_1 (\text{IQ}) + b_2 (\text{Study Time}) + e</math></b>	<b>.80</b>
<b>Reduced: Test Scores = <math>b_0 + b_1 (\text{IQ}) + e</math></b>	<b>.60</b>
<b>Change in R<sup>2</sup> values = <math>\Delta R^2 (\text{Study Time}) = .80 - .60 =</math></b>	<b>.20</b>

### 3. Hypothesis Tests

#### Null Hypothesis

**Study Time does not increase or contribute to the predictive power of the regression model; the variable Study Time does not reduce error in prediction.**

$$\mathbf{H_0: \Delta R^2 (\text{Study Time}) = 0.00}$$

#### Alternative Hypothesis

**Study Time does increase predictive power of regression model.**

$$\mathbf{H_1: \Delta R^2 (\text{Study Time}) \neq 0.00}$$

#### Partial F-test

**A partial F-test is used to test whether  $\Delta R^2$  increase is more than would be expected by chance.**

$$F = \frac{\Delta R^2 (X) / (df_{2reduced} - df_{2full})}{(1 - R_{full}^2) / df_{2full}}$$

$$\mathbf{df_1 = df_{2reduced} - df_{2full},}$$

**and**

$$\mathbf{df_2 = df_{2full}}$$

#### 4. Example with SPSS

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<b>Reading Test Scores</b>	<b>Study Time</b>	<b>IQ</b>	<b>Teacher</b>
<b>85</b>	<b>7</b>	<b>105</b>	<b>Griffin</b>
<b>73</b>	<b>0</b>	<b>95</b>	<b>Griffin</b>
<b>86</b>	<b>5</b>	<b>100</b>	<b>Griffin</b>
<b>81</b>	<b>4</b>	<b>103</b>	<b>Griffin</b>
<b>99</b>	<b>6</b>	<b>113</b>	<b>Moore</b>
<b>93</b>	<b>4</b>	<b>108</b>	<b>Moore</b>
<b>86</b>	<b>2</b>	<b>95</b>	<b>Moore</b>
<b>81</b>	<b>2</b>	<b>100</b>	<b>Moore</b>
<b>77</b>	<b>3</b>	<b>98</b>	<b>Smith</b>
<b>82</b>	<b>2</b>	<b>102</b>	<b>Smith</b>
<b>86</b>	<b>4</b>	<b>110</b>	<b>Smith</b>
<b>91</b>	<b>5</b>	<b>111</b>	<b>Smith</b>

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	<b><math>\Delta R^2</math></b>	<b>df<sub>1</sub></b>	<b>df<sub>2</sub></b>	<b>F</b>
<b>Study Time</b>	<b>.057</b>	<b>1</b>	<b>7</b>	<b>3.16</b>
<b>IQ</b>	<b>.068</b>	<b>1</b>	<b>7</b>	<b>3.80</b>
<b>Teacher</b>	<b>.224</b>	<b>2</b>	<b>7</b>	<b>6.20*</b>

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**\*p<.05**