

Multiple Linear Regression with Categorical and Quantitative Independent Variables

1. Statistical Adjustment – Brief Review

In the previous presentation on Statistical Control and Adjustment the idea of adjustment was illustrated with an example of an experiment with a pretest, posttest, and two groups, control and experimental. The presentation and video can be found here:

PDF: <https://www.bwgriffin.com/gsu/courses/edur8132/notes/reg/Notes-8g-Statistical-Control.pdf>

Video: <https://www.bwgriffin.com/gsu/courses/edur8132/video/regression-control/8g-reg-control.mp4>

You should be familiar with the concept of statistical adjustment. The details presented above will be briefly presented here as a refresher.

Control is the process of examining relations or comparing groups when another variable is held constant. One might ask, for example, what is the expected mean difference between experimental and control groups if they started the experiment with the same level of prior knowledge (i.e., same mean pretest score) as measured by a pretest? Thus, the pretest can be statistically controlled by examining predicted posttest means for select levels of the pretest, i.e., what is the **predicted mean posttest score** for the experimental and control groups if both groups had a mean pretest score of 25%?

Predicted mean posttest scores can be obtained from the regression equation containing a dummy for group and pretest scores; and these predicted means are called **adjusted means**. If there are no mean differences between the two groups, then no adjustment is needed to equate the groups (i.e., hold the groups constant on the pretest). Table A below shows no change from observed to adjusted means since both groups had the same mean pretest score.

Table A: No difference in pretest scores

Group	Pretest M	Observed Posttest M	Adjusted Posttest M
Experimental	32.50	89.50	89.50
Control	32.50	83.50	83.50
Mean Difference =	0.00	6.00	6.00

Suppose, however, that the control group scored lower on the pretest, so the two groups were not equivalent at the beginning of the study. To control for pretest scores, i.e., holding both groups constant on the pretest, it is necessary to adjust posttest means according to starting differences between the groups on the pretest. If the pretest and posttest were positively correlated and if the control group scored lower on the pretest, then the control group's posttests mean will be adjusted higher and the experimental group's posttest mean will be adjusted lower to account for the initial groups' difference. The logic here is similar to a handicap in golf where the weaker player gets benefits in scoring. Table B illustrates this adjustment; the adjustment occurs because of the mean difference in the pretest.

Table B: Control group starts study with lower pretest scores

Group	Pretest M	Observed Posttest M	Direction of Posttest Adjustment	Adjusted Posttest M
Experimental	32.50	89.50	↓	87.70
Control	27.50	83.50	↑	85.30
Mean Difference =	0.00	6.00		2.40

Additional discussion of statistical adjustment, with graphical illustrations, is shown in the ANCOVA presentations.

2. Regression Equation for Quantitative and Categorical Predictors

With both categorical and quantitative predictors, the regression equation remains unchanged except for some interpretational differences for the regression coefficients. The following fictional data, in Table 1, will be used to illustrate the regression analysis and interpretation of the regression equation.

Table 1: Fictional Math Test Scores by Teacher, Student IQ, and Student Motivation

Posttest Math Scores	Teacher	Pretest Math Scores	Math Motivation	Smith	Collins	Brown
70.00	Brown	24.00	7.00	0	0	1
71.00	Brown	25.00	6.00	0	0	1
73.00	Brown	25.00	7.00	0	0	1
74.00	Brown	25.00	6.00	0	0	1
79.00	Smith	26.00	9.00	1	0	0
80.00	Smith	27.00	8.00	1	0	0
82.00	Smith	28.00	9.00	1	0	0
83.00	Smith	29.00	10.00	1	0	0
87.00	Collins	26.00	8.00	0	1	0
88.00	Collins	26.00	11.00	0	1	0
90.00	Collins	27.00	12.00	0	1	0
91.00	Collins	27.00	9.00	0	1	0

Data are available here:

http://www.bwgriffin.com/gsu/courses/edur8132/notes/reg/Notes_8g_fictional_math_scores2.sav

Assume we are interested in learning whether posttest math scores differ by instructor controlling for student math motivation and pretest math scores. The regression equation would be:

$$Y_i = b_0 + b_1\text{Smith}_{1i} + b_2\text{Collins}_{2i} + b_3\text{Pretest}_{3i} + b_4\text{MOTIVATION}_{4i} + e_i, \quad (1)$$

where Smith (1 = in Smith's class, 0 = other) and Collins (1 = in Collin's class, 0 = other) are dummy variables. The SPSS estimates are provided below in Table 2.

Note: Show analysis in both SPSS and JASP.

Table 2: SPSS results for data in Table 1

Model		Coefficients ^a						
		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	28.123	10.633		2.645	.033	2.980	53.266
	Smith	5.804	1.236	.384	4.695	.002	2.881	8.727
	Collins	11.997	1.622	.794	7.398	.000	8.162	15.832
	Pretest_Math_Scores	1.761	.444	.337	3.968	.005	.711	2.810
	Motivation	.046	.320	.012	.144	.890	-.710	.802

a. Dependent Variable: Posttest_Math_Scores

The sample prediction equation for these data is

$$Y' = 28.123 + 5.804(\text{Smith}) + 11.997(\text{Collins}) + 1.761(\text{Pretest}) + 0.046(\text{MOTIVATION}) \quad (2)$$

Since regression equation contains multiple predictors, the coefficients represent partial statistical effects—the statistical association between X_1 and Y controlling for X_2 . This is the same logic discussed earlier with multiple regressions.

Since this above equation contains both qualitative and quantitative predictors, this model is identical to an Analysis of Covariance (ANCOVA) where the two quantitative predictors, Pretest Scores and Motivation, are known as **covariates**. These are control variables used to adjust or equate groups, or to partial the effects of potentially confounding variables.

Interpretation of coefficients remains the same as with previous multiple regression models discussed. One minor difference is that dummy variables now represented the **adjusted mean difference** between groups, adjusted for the statistical effects of the quantitative predictors, both pretest math scores and math motivation.

(1) What is the literal interpretation for $b_0 = 28.123$?

The score 28.123 is the predicted posttest math mean for students Brown's class (the comparison group) who have a score of 0.00 for the math pretest and motivation.

$$\begin{aligned} Y' &= 28.123 + 5.804(\text{Smith}) + 11.997(\text{Collins}) + 1.761(\text{Pretest}) + 0.046(\text{MOTIVATION}) \\ Y' &= 28.123 + 5.804(0) + 11.997(0) + 1.761(0) + 0.046(0) \\ Y' &= 28.123 \end{aligned}$$

(2) What is the literal interpretation for $b_1 = 5.804(\text{Smith})$?

The coefficient $b_1 = 5.804(\text{Smith})$ represents the adjusted mean difference between Smith's and Brown's students controlling for math pretest scores and math motivation scores. Since the value is positive that means Smith's students scored about 5.8 points higher than Brown's students once pretest and motivation are taken into account.

(3) What is the literal interpretation for $b_2 = 11.997(\text{Collins})$?

The coefficient $b_2 = 11.997(\text{Collins})$ represents the adjusted mean difference between Collin's and Brown's students controlling for math pretest scores and math motivation scores. Since the value is positive that means Collin's students scored about 11.99 points higher than Brown's students once pretest and motivation are taken into account.

(4) What is the literal interpretation for $b_3 = 1.761(\text{Pretest Math})$?

The slope for pretest math scores is $b_3 = 1.761$ which means that for each one point increase in pretest math scores, posttest math scores are expected to increase by 1.761 points controlling for teacher and math motivation.

(5) What is the literal interpretation for $b_4 = 0.046(\text{MOTIVATION})$?

The slope for math motivation is $b_4 = 0.046$ which means that for each one point increase in math motivation, posttest math scores are expected to increase by 0.046 points controlling for teacher and pretest math scores.

3. Predicted Values

The observed, unadjusted means for achievement, IQ, and motivation are presented in Table 3 below.

Table 3: Descriptive Statistics for Math Test Scores, IQ, and Motivation by Instructor and Overall

	Math Posttest Scores		Math Pretest		Math Motivation		n
	M	SD	M	SD	M	SD	
Brown	72.00	1.825	24.75	0.500	6.50	0.577	4
Smith	81.00	1.825	26.50	0.577	9.00	0.817	4
Collins	89.00	1.825	27.50	1.291	10.00	1.825	4
Overall	80.667	7.4386	26.25	1.422	8.5000	1.8829	12

A benefit of the inclusion of covariates, or quantitative predictors, when groups are compared is statistical adjustment of group means and mean differences. Statistical adjustment may provide some insight into how the groups may perform on the DV if each group scored the same on the covariates. Statistical adjustment is the result of the partialing effects of regression.

As noted above, the prediction equation for this model is:

$$Y' = 28.123 + 5.804(\text{Smith}) + 11.997(\text{Collins}) + 1.761(\text{Pretest}) + 0.046(\text{MOTIVATION}) \quad (2)$$

To obtain predicted means, or adjusted means, one must substitute the mean value of the covariates into the regression equation. For the current example, these values would be used:

Mean of Math Pretest = 26.25

Mean of Motivation = 8.50

$$Y' = 28.123 + 5.804(\text{Smith}) + 11.997(\text{Collins}) + 1.761(\text{Pretest}=26.25) + 0.046(\text{MOTIVATION}=8.50)$$

(1) What is the predicted mean (adjusted mean) for Brown's class?

$$Y' = 28.123 + 5.804(\text{Smith}) + 11.997(\text{Collins}) + 1.761(\text{Pretest}=26.25) + 0.046(\text{MOTIVATION}=8.50)$$

$$Y' = 28.123 + 5.804(0) + 11.997(0) + 1.761(26.25) + 0.046(8.50)$$

$$Y' = 28.123 + 5.804(0) + 11.997(0) + 1.761(26.25) + 0.046(8.50)$$

$$Y' = 28.123 + 5.804(0) + 11.997(0) + 46.2263 + 0.391$$

$$Y' = 74.7403$$

(2) What is the predicted mean (adjusted mean) for Smith's class?

$$Y' = 28.123 + 5.804(\text{Smith}) + 11.997(\text{Collins}) + 1.761(\text{Pretest}=26.25) + 0.046(\text{MOTIVATION}=8.50)$$

$$Y' = 28.123 + 5.804(1) + 11.997(0) + 1.761(26.25) + 0.046(8.50)$$

$$Y' = 28.123 + 5.804(1) + 11.997(0) + 1.761(26.25) + 0.046(8.50)$$

$$Y' = 28.123 + 5.804(1) + 11.997(0) + 46.2263 + 0.391$$

$$Y' = 80.5443$$

(3) What is the predicted mean (adjusted mean) for Collin's class?

$$Y' = 28.123 + 5.804(\text{Smith}) + 11.997(\text{Collins}) + 1.761(\text{Pretest}=26.25) + 0.046(\text{MOTIVATION}=8.50)$$

$$Y' = 28.123 + 5.804(0) + 11.997(1) + 1.761(26.25) + 0.046(8.50)$$

$$Y' = 28.123 + 5.804(0) + 11.997(1) + 1.761(26.25) + 0.046(8.50)$$

$$Y' = 28.123 + 5.804(0) + 11.997(1) + 46.2263 + 0.391$$

$$Y' = 86.7373$$

Table 4: Observed Means and Adjusted Means

Instructor	Observed Mean	Adjusted Mean
Brown	72.00	74.7303
Smith	81.00	80.5443
Collins	89.00	86.7373

(4) Why do the adjusted means (estimated means or predicted means) differ from the observed means? Why does this difference occur?

If the groups have different means for a covariate, or all covariates, then those differences are statistically controlled by producing predicted means (adjusted means) that hold constant levels of the covariates by using the same covariate mean for each group. See Table 3 above and note that the Pretest Math mean for both Smith (26.50) and Collins (27.50) was higher than the overall Pretest Math mean (26.25). For Brown (24.75), the Pretest Math mean was lower than overall mean. So, when Pretest Math scores are held constant, i.e., the regression equation uses the overall Pretest Math mean to make predictions, then the adjusted mean for Brown is higher than Brown's observed mean while both Smith and Collins have adjusted means that are lower than their observed means. Thus, the regression equation helps to statistically equate groups on a covariate with pre-existing group differences by using one mean – the overall – instead of individual group means on the covariate and this tends to adjust the DV to compensate for initial differences on the covariate.

4. Model Fit and Model Statistical Inference

The usual measures of fit and inference for the overall models continues to apply here. Overall model fit is assessed with R^2 , adjusted R^2 , mean squared variance, and standard error of estimate. The null for the overall model is

$$H_0: R^2 = 0.00$$

which means the model predictors do not predict or explain any variation in the DV, or alternatively,

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.00$$

which means that none of predictors' coefficients differ from 0.00 so they are all unrelated to the DV. The overall model null is tested with an F-test, as shown previously.

If the overall model null is rejected, then move to the predictor and/or individual regression coefficients to assess significance. For categorical variables with more than two categories, use the F-test to test $H_0: \Delta R^2 = 0.00$ to assess the contribution of that variable to the model. For covariates or categorical variables with only two categories, the t-ratio for the coefficient is suitable to test the null that $H_0: \beta_j = 0.00$ (or use the confidence interval to test whether 0.00 lies within the interval).

5. Global Effects, ΔR^2 , and the Partial F Test of ΔR^2

Statistical inference regarding the global effect, as measured by $\Delta R^2(X_k)$, continues to hold here. To illustrate, the overall statistical effect of instructor upon math scores will be tested. The reduced model contains only Math Pretest and Motivation:

$$Y_i = b_0 + b_3 \text{Pretest}_{3i} + b_4 \text{MOTIVATION}_{4i} + e_i, \quad (3)$$

and the full model contains Pretest, Motivation, and Instructor dummy variables:

$$Y_i = b_0 + b_1 \text{Smith}_{1i} + b_2 \text{Collins}_{2i} + b_3 \text{Pretest}_{3i} + b_4 \text{MOTIVATION}_{4i} + e_i, \quad (4)$$

The null hypothesis for the instructor statistical effect follows:

$$H_0: \Delta R^2(\text{instructor}) = \Delta R^2(\text{Smith, Collins}) = 0.00.$$

This hypothesis can be tested by hand or in SPSS.

Table 5

Model	R ²	Regression df	Error df
Y' = reduced model 3 above (pretest, motivation)	.862	2	9
Y' = full model 4 above (+ teacher)	.986	4	7

$\Delta R^2(\text{Instructor}) = .986 - .862 = .124$
 $\Delta R^2 \text{ df}_1 = 4 - 2 = 2$
 $\Delta R^2 \text{ df}_2 = 7$ (smaller df)

If calculated by hand, the F ratio is calculated is

$$F = \frac{\Delta R^2 / (df_{2\text{reduced}} - df_{2\text{full}})}{(1 - R_{\text{full}}^2) / df_{2\text{full}}} = \frac{.124 / (9 - 7)}{(1 - .986) / 7} = \frac{.0175}{.002} = 31.00$$

The df for this test are:

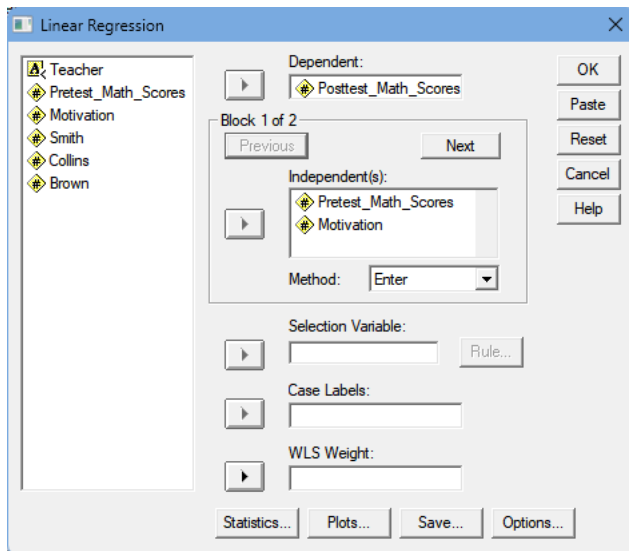
$$df_1 = df_{2\text{reduced}} - df_{2\text{full}} = 9 - 7 = 2, \text{ and}$$

$$df_2 = df_{2\text{full}} = 7.$$

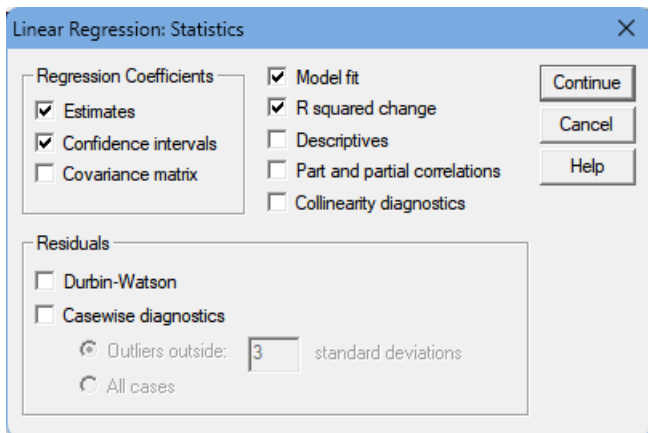
The critical F at $\alpha = .05$ would be 4.74. Since $F = 31.00$ is greater than critical $F = 4.74$, reject H_0 and conclude teachers do contribute to variability in math posttest scores in these data.

In SPSS

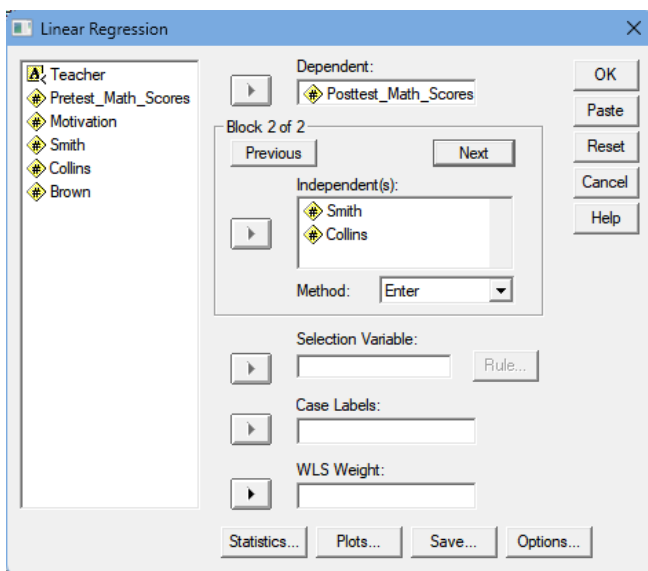
1. Choose Regression, enter Math Posttest Scores in the Dependent box
2. Enter Pretest and Motivation in Independents box



3. Click on Statistics, R-square Change, Continue



4. Click Next, then enter Smith and Collins dummy variables in IV box
5. Click Ok to obtain results



See image below for SPSS results showing test of global effect ΔR^2 (Instructor).

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.928 ^a	.862	.831	3.05790	.862	28.046	2	9	.000
2	.993 ^b	.986	.978	1.11106	.124	30.587	2	7	.000

a. Predictors: (Constant), Motivation, Pretest_Math_Scores
 b. Predictors: (Constant), Motivation, Pretest_Math_Scores, Smith, Collins

Recall that the hand-calculated F ratio was $F = 31.00$. The difference between this value and the value of 30.587 reported above is due to rounding error (level of precision with which R^2 is reported in JASP and SPSS).

In JASP

1. Choose Regression, Linear Regression, enter Math Posttest Scores in the Dependent box
2. Enter Pretest and Motivation in the Covariates box, and then enter Teacher in the Factors box (to enter the Factors box, Teacher must be a nominal variable – see video presentation here to see how to change or confirm)

Note: Show data check on Teacher to ensure nominal variable specified in JASP.

3. Click on Statistics, then place mark next to R-square Change and Confidence Intervals (set at 95% by default)

Linear Regression

```
jaspRegression::RegressionLinear(  
  data = NULL,  
  version = "0.18.3",  
  formula = "Posttest_Math_Scores ~ Teacher + Motivation + Pretest_Math_Scores",  
  covariates = list("Pretest_Math_Scores", "Motivation"),  
  coefficientBootstrap = FALSE,  
  coefficientBootstrapSamples = 5000  
)
```

Ctrl + Enter to apply

Dependent Variable: Posttest_Math_Scores

Method: Enter

Covariates: Pretest_Math_Scores, Motivation

Factors: Teacher

WLS Weights (optional)

Model

Statistics

Coefficients

- Estimates
- From 5 bootstraps
- Confidence intervals 95.0 %
- Covariance matrix
- Vovk-Sellke maximum p-ratio
- Model fit
- R squared change
- Descriptives
- Part and partial correlations
- Collinearity diagnostics

4. Click Model, then add Pretest and Motivation to the null model by placing market next to both. This identifies the remaining variable, Teachers, as the second step model to produce the ΔR^2 value.

Model

Components

- Pretest_Math_Sc
- Motivation
- Teacher

Model Terms

- Pretest_Math_Scores
- Motivation
- Teacher

Add to null model

-
-
-

5. Review results

See image below for JASP results showing test of global effect $\Delta R^2(\text{Instructor})$.

Model Summary - Posttest_Math_Scores									
Model	R	R ²	Adjusted R ²	RMSE	R ² Change	F Change	df1	df2	p
H ₀	0.928297104	0.861735512	0.831010071	3.057903871	0.861735512	28.046318134	2	9	0.000135893
H ₁	0.992876238	0.985803224	0.977690781	1.111055116	0.124067712	30.587014427	2	7	0.000346878

Note. Null model includes Pretest_Math_Scores, Motivation

6. Inferential Procedures for Regression Coefficients

For variables that take one column, or vector, of data (such as the quantitative predictors), the t-ratio of b/se is sufficient for hypothesis testing. This is covered elsewhere and will be briefly reviewed here in the video.

SPSS

Coefficients ^a								
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-24.610	21.858		-1.126	.289	-74.056	24.836
	Pretest_Math_Scores	3.656	1.004	.699	3.640	.005	1.384	5.928
	Motivation	1.096	.759	.277	1.445	.182	-.620	2.812
2	(Constant)	28.123	10.633		2.645	.033	2.980	53.266
	Pretest_Math_Scores	1.761	.444	.337	3.968	.005	.711	2.810
	Motivation	.046	.320	.012	.144	.890	-.710	.802
	Smith	5.804	1.236	.384	4.695	.002	2.881	8.727
	Collins	11.997	1.622	.794	7.398	.000	8.162	15.832

a. Dependent Variable: Posttest_Math_Scores

JASP

Coefficients								
Model		Unstandardized	Standard Error	Standardized ^a	t	p	95% CI	
							Lower	Upper
H ₀	(Intercept)	-24.609958506	21.857928650		-1.125905336	0.289331812	-74.056028366	24.83611353
	Pretest_Math_Scores	3.655601660	1.004389078	0.698930696	3.639627052	0.005404022	1.383515712	5.927687607
	Motivation	1.096127248	0.758638162	0.277462003	1.444861730	0.182396693	-0.620031506	2.812286001
H ₁	(Intercept)	28.122699387	10.633032964		2.644842679	0.033189257	2.979571771	53.265827002
	Pretest_Math_Scores	1.760736196	0.443739894	0.336642963	3.967946584	0.005406467	0.711458082	2.810014311
	Motivation	0.046012270	0.319748625	0.011647057	0.143901385	0.889633498	-0.710073083	0.802097623
	Teacher (Collins)	11.996932515	1.621671665		7.397880086	0.000149692	8.162288368	15.831576662
	Teacher (Smith)	5.803680982	1.236085674		4.695209325	0.002220927	2.880802821	8.726559143

^a Standardized coefficients can only be computed for continuous predictors.

7. Pairwise Comparisons and Multiple Comparisons Among IV Categories

For categorical (qualitative) predictors with more than two categories, such as instructor in the current example, one may need to perform pairwise comparisons to identify statistical difference if the Global Effect test is statistically significant, i.e., $H_0: \Delta R^2(\text{instructor}) = 0.00$ is rejected.

Use either the Bonferroni, Scheffé, or Tukey HSD, as illustrated before. One must perform comparisons among the adjusted mean differences, which are provided by the regression coefficients. With the current example regression equation:

$$Y_i = b_0 + b_1\text{Smith}_{1i} + b_2\text{Collins}_{2i} + b_3\text{Pretest}_{3i} + b_4\text{MOTIVATION}_{4i} + e_i, \quad (1)$$

here

b_1 = adjusted mean difference in math scores between Smith's class and Brown's class; and

b_2 = adjusted mean difference in math scores between Collin's class and Brown's class.

SPSS Results

Coefficients ^a								
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	28.123	10.633		2.645	.033	2.980	53.266
	Smith	5.804	1.236	.384	4.695	.002	2.881	8.727
	Collins	11.997	1.622	.794	7.398	.000	8.162	15.832
	Pretest_Math_Scores	1.761	.444	.337	3.968	.005	.711	2.810
	Motivation	.046	.320	.012	.144	.890	-.710	.802

a. Dependent Variable: Posttest_Math_Scores

b_1 = Smith vs. Brown = 5.804 (se = 1.236)

b_2 = Collin vs. Brown = 11.997 (se = 1.622)

The last comparison is between Smith and Collins, so re-run the regression and make Collins the reference group by removing the Collins dummy variable and including the Brown dummy variable:

$$Y_i = b_0 + b_1\text{Smith}_{1i} + b_2\text{Brown}_{2i} + b_3\text{Pretest}_{3i} + b_4\text{MOTIVATION}_{4i} + e_i, \quad (4)$$

SPSS Results

Coefficients ^a								
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	40.120	11.774		3.408	.011	12.280	67.960
	Pretest_Math_Scores	1.761	.444	.337	3.968	.005	.711	2.810
	Motivation	.046	.320	.012	.144	.890	-.710	.802
	Smith	-6.193	.917	-.410	-6.755	.000	-8.361	-4.025
	Brown	-11.997	1.622	-.794	-7.398	.000	-15.832	-8.162

a. Dependent Variable: Posttest_Math_Scores

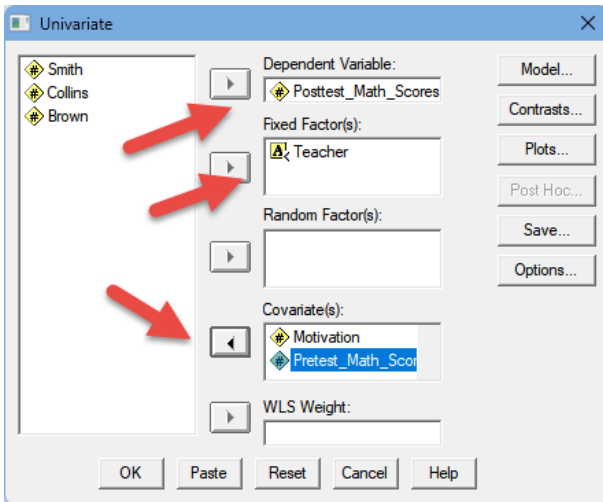
b_1 = Smith vs. Collin = -6.193 (se = 0.917)

Standard errors for each of the regression coefficients are reported by SPSS, and calculation of the confidence intervals for the adjusted mean differences are performed as illustrated previously. To control for inflation of the familywise Type 1 error rate, one may use ANOVA commands to obtain the corrected confidence intervals and p-values or calculate them by hand. The ANOVA approach is illustrated below since it is both easier and faster.

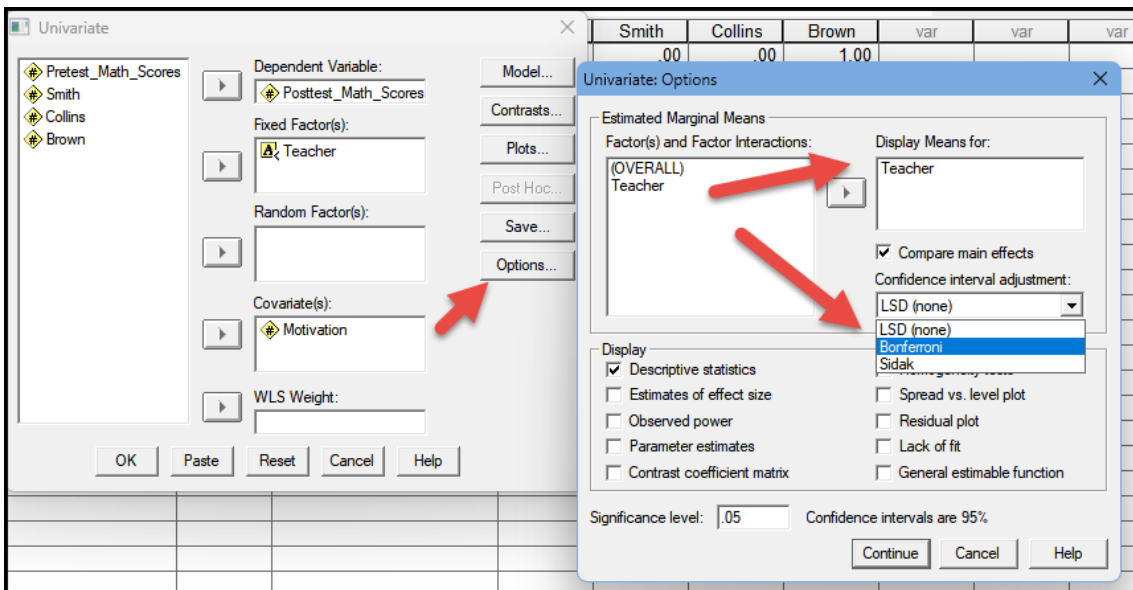
Multiple Comparisons using ANOVA/ANCOVA in SPSS and JASP

SPSS

1. Analyze, General Linear Model, Univariate
2. Move Posttest to Dependent box, Teacher to Fixed Factor box, and Pretest and Motivation to Covariates box.



3. Click on Options, then move Teacher to Display Means box, then select Bonferroni on the pull-down menu. Click Continue then OK.



4. The pairwise comparison table appears below using the Bonferroni procedure. Neither the Scheffé nor Tukey procedure are available in the SPSS Univariate command with my version of SPSS.

Pairwise Comparisons

Dependent Variable: Posttest_Math_Scores

(I) Teacher	(J) Teacher	Mean Difference (I-J)	Std. Error	Sig. ^a	95% Confidence Interval for Difference ^a	
					Lower Bound	Upper Bound
Brown	Collins	-11.997*	1.622	.000	-17.069	-6.925
	Smith	-5.804*	1.236	.007	-9.670	-1.938
Collins	Brown	11.997*	1.622	.000	6.925	17.069
	Smith	6.193*	.917	.001	3.326	9.061
Smith	Brown	5.804*	1.236	.007	1.938	9.670
	Collins	-6.193*	.917	.001	-9.061	-3.326

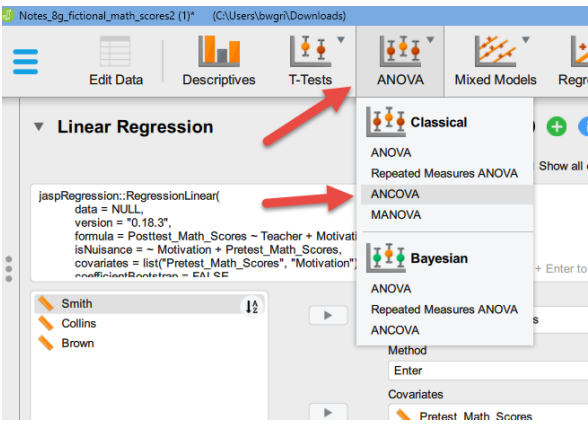
Based on estimated marginal means

*. The mean difference is significant at the .05 level.

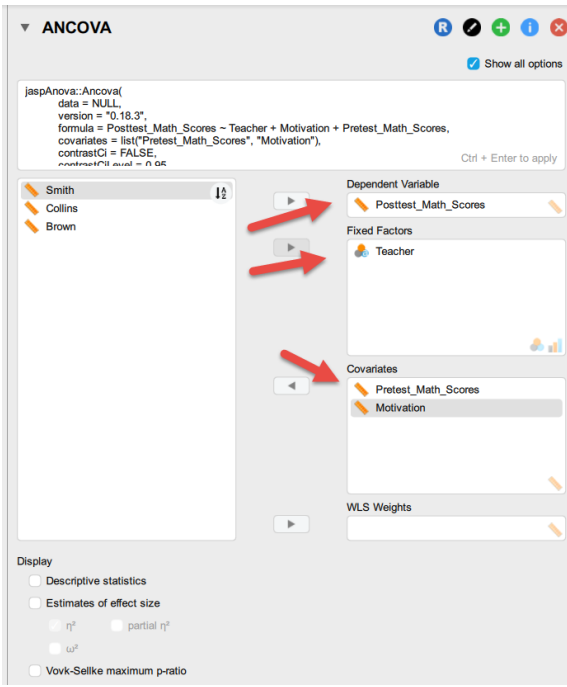
a. Adjustment for multiple comparisons: Bonferroni.

JASP

1. ANOVA, ANCOVA



2. Move Posttest to Dependent box, Teacher to Fixed Factor box, and Pretest and Motivation to Covariates box.



3. Click on Post Hoc Tests, then move Teacher to right box, then select Tukey under Correction procedure, then select Confidence Interval (95% by default).

The screenshot shows the JASP software interface for Post Hoc Tests. The 'Post Hoc Tests' section is expanded, showing two boxes for variable selection. A red arrow points from the left box to the 'Teacher' variable in the right box. Below the boxes, the 'Type' section has 'Standard' selected. The 'Correction' section has 'Tukey' selected. The 'Display' section has 'Confidence intervals' selected with a value of 95.0%. A red arrow points to the 'Confidence intervals' checkbox. At the bottom, there is a 'Descriptives Plots' section.

4. The Post Hoc Tests pairwise comparison table appears below using the Tukey HSD procedure. None of the other procedures offered by JASP provides corrected confidence intervals.

Post Hoc Tests

Standard

Post Hoc Comparisons - Teacher

		95% CI for Mean Difference			SE	t	P _{Tukey}
		Mean Difference	Lower	Upper			
Brown	Collins	-11.996932515	-16.772850238	-7.221014793	1.621671665	-7.397880086	0.000378288
	Smith	-5.803680982	-9.444025450	-2.163336513	1.236085674	-4.695209325	0.005464399
Collins	Smith	6.193251534	3.493043665	8.893459403	0.916860558	6.754845627	0.000664256

Note. P-value and confidence intervals adjusted for comparing a family of 3 estimates (confidence intervals corrected using the tukey method).

Below Table 5 shows both sets of confidence intervals from SPSS and JASP. Note that JASP does not provide a complete pairwise table like SPSS, so the direction of comparisons did not fit the direction I selected when initially constructing this table. If one wanted a different direction of comparison, i.e. Smith vs. Brown rather than Brown vs. Smith, one could easily change the direction of each number by multiplying by -1. For example, to convert Brown vs. Smith to Smith vs. Brown:

Original Mean Difference = -5.801 becomes $-1 * -5.801 = 5.801$

Original Lower CI = -9.444 becomes $-1 * -9.444 = 9.444$

Original Upper CI = -2.163 becomes $-1 * -2.163 = 2.163$

With conversion one now has the Smith vs Brown comparisons
 mean difference = 5.804 with Tukey CI of 2.163, 9.444

Table 5: Multiple Comparisons with Adjusted Confidence Intervals

Comparison	Adjusted mean difference	se	Bonferroni Adjusted Confidence Interval	Tukey HSD CI
SPSS				
Smith vs. Brown	5.804	1.236	1.935, 9.670	---
Collins vs. Brown	11.997	1.622	6.925, 17.069	---
Smith vs. Collins	-6.193	0.917	-9.061, -3.326	---
JASP				
Brown vs. Smith	-5.803	1.236	---	-9.444, -2.163
Brown vs. Collins	-11.997	1.622	---	-16.773, -7.221
Collins vs. Smith	6.193	0.917	---	3.493, 8.893

APA styled results provided below.

7. APA Style Results

Table 6: Descriptive Statistics and Correlations Among Posttest and Pretest Math Scores, Motivation, and Teachers

Variable	Correlations				
	Posttest Scores	Pretest Scores	Motivation	Smith	Collins
Posttest Scores	---				
Pretest	.911*	---			
Motivation	.811*	.764*	---		
Smith	.030	.130	.196	---	
Collins	.827*	.649*	.588*	-.500	---
Mean	80.667	26.250	8.50	0.33	0.33
SD	7.438	1.422	1.88	0.49	0.49

Note. Smith (1 = students in Smith's class, 0 = others) and Collins (1 = students in Collin's class, 0 = others) are dummy variables; n = 12.

*p<.05.

Table 7: Regression of Posttest Math Scores on Teachers, Pretest Scores, and Motivation

Variable	b	se	ΔR^2	95%CI	F	t
Pretest	1.760	0.443	0.031	0.711, 2.810		3.967*
Motivation	0.046	0.319	0.000	-0.710, 0.802		0.143
Teacher			0.124		30.587*	
Smith	5.803	1.236		2.880, 8.726		4.695*
Collins	11.997	1.621		8.162, 15.831		7.397*
Intercept	28.122	10.633		2.979, 53.265		2.644*

Note. $R^2 = .985$, adj. $R^2 = .977$, $F_{4,7} = 121.517^*$, $MSE = 1.234$, $n = 12$. ΔR^2 represents the squared semi-partial multiple correlation or the increment in R^2 due to adding the respective variable. Smith (1 = students in Smith's class, 0 = others) and Collins (1 = students in Collin's class, 0 = others) are dummy variables.

*p<.05.

Table 8: Comparisons of Adjusted Mean Math Scores Among Instructors

Comparisons	Estimated Adjusted Mean Difference	Standard Error of Difference	Bonferroni Adjusted .95CI
Smith vs. Brown	5.804*	1.236	1.935, 9.670
Collins vs. Brown	11.997*	1.622	6.925, 17.069
Smith vs. Collins	-6.193*	0.917	-9.061, -3.326

Note. Math score comparisons adjusted based upon IQ and Motivation.

*p<.05, where p-values are adjusted using the Bonferroni method.

Math pretest scores are positively related to math posttest scores, and there are statistical differences in mean math scores among instructors. Motivation, once instructor and pretest scores are controlled, does not appear to be related to math scores. All pairwise comparisons were performed, and all were statistically significant at the 5% level using the Bonferroni adjustment. Students in Collins' class performed best, those in Smith's performed worst, those in Brown's class scored between Collins' and Smith's classes.