Multiple Linear Regression with Categorical Independent Variables

1. Regression Equation

The regression equation remains the same as before except for the addition of an extra predictor. For example, suppose we have the following data with two predictors, student sex and teacher:

Table 1

Math Scores	Student Sex	Teacher	Math Scores	Student Sex	Teacher	Math Scores	Student Sex	Teacher
72	F	Gunther	74	F	Bryan	78	F	Marijke
73	F	Gunther	75	F	Bryan	79	F	Marijke
74	F	Gunther	76	F	Bryan	80	F	Marijke
76	M	Gunther	80	M	Bryan	83	M	Marijke
77	M	Gunther	81	M	Bryan	84	M	Marijke
78	M	Gunther	82	M	Bryan	85	M	Marijke

These data may be downloaded from the following link. Note the structure of the data in SPSS and JASP – all data in three columns, one for scores, one for sex, and one for teacher. Dummy variables are then created to identify group membership for sex or teacher.

http://www.bwgriffin.com/gsu/courses/edur8132/notes/math_scores.sav

The sample regression equation takes this form:

$$Y_i = b_0 + b_1 Male_{1i} + b_2 Bryan_{2i} + b_3 Marijke_{3i} + e_i, \quad (1)$$

Regression coefficients maintain interpretations as learned previously, except now we adding the notion of statistical control since there is more than one predictor.

b_1 = since Male will be the dummy variable, b_1 is mean difference in math scores between males and females controlling for teacher.

b_2 = since the dummy variable Bryan identifies the teacher, b_2 is the mean difference in math scores between Bryan and Gunther (the omitted or reference teacher) controlling for student sex.

b_3 = and the dummy variable for teacher Marijke, b_3 is the mean difference in math scores between Marijke and Gunther controlling for student sex.

b_0 = predicted value of Y, Y', when IV equal zero; note that when dummy variables are in the equation, values of 0 for dummy represent the omitted group; literal interpretation for b_0 in this equation:

b_0 is the predicted mean math score for females in Gunther's class.

2. Predicted Values and Errors

As before, predicted values are obtained using the equation:

$$Y' = b_0 + b_1 Male_{1i} + b_2 Bryan_{2i} + b_3 Marijke_{3i} \quad (2)$$

Predicted means are often called **marginal means** or **adjusted means** in software such as SPSS and JASP and covariates or other categorical variables are usually set at their mean values when calculating marginal means.

Residuals are obtained by

$e_i = Y - Y'$.

For the current data the following results are obtained:

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$

(1) What is the predicted mean score for Females in Gunther's class?

Females in Gunther's class is the comparison, or omitted, group in the regression equation so the intercept, b_0, represents their predicted mean score = 72.5

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (0) + 3.00\ (0) + 6.50\ (0)$
$Y' = 72.5$

(2) What is the predicted mean score for Males in Gunther's class?

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (1) + 3.00\ (0) + 6.50\ (0)$
$Y' = 72.5 + 5.00$
$Y' = 77.5$

(3) What is the predicted mean score for Females in Bryan's class?

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (0) + 3.00\ (1) + 6.50\ (0)$
$Y' = 72.5 + 3.00$
$Y' = 75.5$

(4) What is the predicted mean score for Males in Bryan's class?

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (1) + 3.00\ (1) + 6.50\ (0)$
$Y' = 72.5 + 5.00 + 3.00$
$Y' = 80.5$

(5) What is the predicted mean score for Females in Marijke's class?

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (0) + 3.00\ (0) + 6.50\ (1)$
$Y' = 72.5 + 6.50$
$Y' = 79$

(6) What is the predicted mean score for Males in Marijke's class?

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (1) + 3.00\ (0) + 6.50\ (1)$
$Y' = 72.5 + 5.00 + 6.50$
$Y' = 84$

(7a) What is the estimated student sex difference in math holding constant teacher?

$Y' = b_0 + b_1 Male_{1i} + b_2 Bryan_{2i} + b_3 Marijke_{3i}$ (2)
$Y' = 72.5 + \mathbf{5.00\ (Male)} + 3.00\ (Bryan) + 6.50\ (Marijke)$
$b_1 = 5.00$

(7b) Is this sex difference the same for all teachers? How does the average/estimated difference compare with actual?

Yes, the same, because there is no interaction term in the regression equation to model possible changes in sex differences across teacher. An interaction term would allow the difference of 5.00 to vary across teachers.

(8) What are the estimated teacher differences in math holding constant student sex?

Bryan vs. Gunther	$= b_2 Bryan_{2i}$
Marijke vs. Gunther	$= b_3 Marijke_{3i}$
Bryan vs. Marijke	= must set Marijke as the comparison teacher and include Gunther dummy to estimate coefficient, but it should be the difference between their coefficient estimates, i.e., $b_2 – b_3 = 3.00 – 6.50 = -3.50$

3. Predicted Values Holding Constant One IV

If one wished to obtain the **predicted means**, or **adjusted means** or **marginal means**, for each teacher controlling for sex – not predicting means separately for males and females, but instead holding constant sex—one must include sex in the regression equation but instead of using the scores 0, 1, one instead using the mean value of the dummy variable for sex. Thus, as noted above, predicted means are usually done by using mean scores for each covariate and grouping variable. This will be explained and illustrated in two future presentations: regression with both quantitative and categorical variables and in ANCOVA.

In this example, since sex is evenly divided, the mean for the sex dummy variable is M = 0.50, so **predicted means** (**marginal means**, or **adjusted means**) are obtained by using the male dummy variable mean as the multiplier for Male rather than the dummy code of 1.

$Y' = b_0 + b_1 Male_{1i} + b_2 Bryan_{2i} + b_3 Marijke_{3i}$
$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ \mathbf{(0.50)} + 3.00\ (Bryan) + 6.50\ (Marijke)$

(9) What is the predicted mean score for Gunther's class, holding constant sex?

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (0.50) + 3.00\ (0) + 6.50\ (0)$
$Y' = 72.5 + 5.00\ (0.50)$
$Y' = 75$

(10) What is the predicted mean score for Bryan's class, holding constant sex?

$Y' = 72.5 + 5.00\ (Male) + 3.00\ (Bryan) + 6.50\ (Marijke)$
$Y' = 72.5 + 5.00\ (0.50) + 3.00\ (1) + 6.50\ (0)$
$Y' = 72.5 + 5.00\ (0.50) + 3.00\ (1)$
$Y' = 78$

(11) What is the predicted mean score for Marijke's class, holding constant sex?

Y' = 72.5 + 5.00 (Male) + 3.00 (Bryan) + 6.50 (Marijke)
Y' = 72.5 + 5.00 (0.50) + 3.00 (0) + 6.50 (1)
Y' = 72.5 + 5.00 (0.50) + 6.50 (1)
Y' = 81.5

The values calculated above are also produced in JASP and SPSS; see results below.

JASP marginal means

Marginal Means

Marginal Means - teacher

teacher	Marginal Mean	95% CI for Mean Difference Lower	Upper	SE
Bryan	78.000	77.094	78.906	0.423
Gunther	75.000	74.094	75.906	0.423
Marijke	81.500	80.594	82.406	0.423

SPSS marginal means

Estimated Marginal Means

teacher

Estimates

Dependent Variable: math

teacher	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
Bryan	78.000	.408	77.111	78.889
Gunther	75.000	.408	74.111	75.889
Marijke	81.500	.408	80.611	82.389

4. Overall Mode Fit and Statistical Inference

The usual statistics apply for overall model fit (R^2, adjusted R^2, MSE, SEE) and the testing of model fit (F-value). If the F ratio for the model is significant (testing Ho: $R^2 = 0.00$) then that means more variance in the DV (math scores) is being predicted than expected by chance. It also implies that at least two group means are significantly different, either the sex means or two of the teachers' means. Use IV coefficient testing to identify which variables are significant.

JASP results for R^2 and model fit

Model Summary - math ▼

Model	R	R^2	Adjusted R^2	RMSE
H_0	0.000	0.000	0.000	3.869
H_1	0.706	0.499	0.432	2.915

ANOVA

Model		Sum of Squares	df	Mean Square	F	p
H_1	Regression	127.000	2	63.500	7.471	0.006
	Residual	127.500	15	8.500		
	Total	254.500	17			

Note. The intercept model is omitted, as no meaningful information can be shown.

5. Individual IV Statistical Inference

As before, each regression coefficient is tested with a t-ratio (b/se = t). However, coefficient t-ratios are insufficient to assess the contribution of a categorical variable with more than two categories. The **global effect** (overall statistical effect on the regression model) of a categorical IV with more than two categories is assessed by the F-test of the $\Delta R^2(X_k)$ contribution to the model. For categorical variables this null is Ho: $\Delta R^2(X_k) = 0.00$ or Ho: category group means are all equal.

Current Example: Teacher Global Effect:

Table 2

Model	R^2	Regression df	Error df
$Y' = b_0 + b_1Male_{1i}$	.442	1	16
$Y' = b_0 + b_1Male_{1i} + b_2Bryan_{2i} + b_3Marijke_{3i}$	.941	3	14
ΔR^2(Teacher) =	.941-.442 = .499	ΔR^2 df_1 = 3-1 = 2	ΔR^2 df_2 = 14 (smaller df)

In SPSS:
1. Choose Regression, enter Math in the Dependent box
2. Enter Male in Independents box, then click on Statistics->R-square Change->Continue
3. Click Next, then enter Bryan and Marijke dummy variables in IV box
4. Click Ok

The image below shows SPSS results testing the global effect ΔR^2(Teacher).

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.665[a]	.442	.407	2.97909	.442	12.676	1	16	.003
2	.970[b]	.941	.928	1.03510	.499	59.267	2	14	.000

a. Predictors: (Constant), Male
b. Predictors: (Constant), Male, Marijke, Bryan

ΔR2 = .499
F ratio = 59.267
df 1 = 2
df 2 = 14
p-value for F = .000 (reject Ho)

Alternatively, one can use the TEST function in SPSS syntax, i.e., /TEST = (MALE) (Bryan Marijke). See the presentation notes and video on calculating ΔR^2. Results presented below.

ANOVA[c]

Model			Sum of Squares	df	Mean Square	F	Sig.	R Square Change
1	Subset Tests	Male	112.500	1	112.500	105.000	.000[a]	.442
		Bryan, Marijke	127.000	2	63.500	59.267	.000[a]	.499
	Regression		239.500	3	79.833	74.511	.000[b]	
	Residual		15.000	14	1.071			
	Total		254.500	17				

a. Tested against the full model.
b. Predictors in the Full Model: (Constant), Marijke, Male, Bryan.
c. Dependent Variable: math

In JASP we can use the H_0 and H_1 models to create steps of entry for predictors. JASP results shown below match those produced by SPSS.

Model Summary - math

Model	R	R^2	Adjusted R^2	RMSE	R^2 Change	F Change	df1	df2	p
H_0	0.665	0.442	0.407	2.979	0.442	12.676	1	16	0.003
H_1	0.970	0.941	0.928	1.035	0.499	59.267	2	14	< .001

Note. Null model includes sex

6. Pairwise Comparisons Among IV Categories

Typically, with multinomial predictors, it is useful to provide a table of pairwise comparisons. Procedures to obtain pairwise comparisons were presented in the previous video on regression with a multinomial variable. Two approaches were used: (a) estimate separate regression equations with different groups serving the role of comparison groups, or (b) use ANOVA commands to obtain all pairwise comparisons automatically with post hoc procedures.

One must also control for familywise Type 1 error inflation when performing multiple comparisons. Pairwise comparisons in the context of regression, or ANOVA, when a second, or more, predictor is present means pairwise comparisons are performed on **adjusted means** (i.e., **predicted means** or **marginal means**). For this example, interest lies in determining whether math means differ among teachers after controlling for student sex.

6a. Regression Model Approach

1. Estimate teacher differences in regression. To obtain all mean differences you will have to change reference groups in the regression equation, for example:

$Y' = b_0 + b_1 Male_{1i} + b_2 Bryan_{2i} + b_3 Marijke_{3i}$

b_2 = Bryan vs. Gunther mean difference
b_3 = Marijke vs. Gunther mean difference

How to get Bryan vs. Marijke mean difference? Rerun regression with Marijke as the omitted, referenced teacher:

$Y' = b_0 + b_1 Male_{1i} + b_2 Bryan_{2i} + b_3 Gunther_{3i}$

Now, b_2 = Bryan vs. Marijke mean difference.

2. Find standard errors (se) for each mean difference
3. Find appropriate Bonferroni or Scheffé critical t-value
4. Calculate CI for each mean difference, e.g,

Upper CI: b2 + se * critical t
Lower CI: b2 – se * critical t

For the current example Bonferroni critical t = 2.709 (with comparisons = 3 and df = 14). For the Bryan vs. Gunther comparison (b2 = 3.00, se = 0.598), the 95% Bonferroni CI is

Upper CI: 3.00 + 0.598 * 2.709 = 4.62
Lower CI: 3.00 – 0.598 * 2.709 = 1.38

Below is a table showing complete results (only one adjusted CI needed; four are provided to show differences in CI among Tukey, Bonferroni, and Scheffé procedures.

Table 3

Comparison	Mean Difference	S.E. of Difference	Bonf. Adj. 95% CI - hand cal.	Bonf. Adj. 95% CI -SPSS	Tukey Adj. 95% CI - SPSS, JASP	Scheffé 95% CI - SPSS
Bryan vs. Gunther	3.00*	0.598	1.38, 4.62	1.395, 4.604	1.459, 4.54	1.39, 4.609
Marijke vs. Gunther	6.50*	0.598	4.88, 8.12	4.895, 8.104	4.959, 8.04	4.89, 8.109
Bryan vs. Marijke	-3.50*	0.598	-5.12, -1.88	-5.104, -1.895	-5.04, -1.959	-5.109, -1.89

*p<.05, where p-values are adjusted using the Bonferroni method.

6b. ANOVA Approach

Since software typically provides multiple comparisons as an option in ANOVA, using the ANOVA option is the more efficient means to obtain multiple comparisons with adjusted confidence intervals.

Both approaches are illustrated in the video. Below are results from both SPSS and JASP.

SPSS Multiple Comparisons from Univariate ANOVA command.

Multiple Comparisons

Dependent Variable: math

	(I) teacher	(J) teacher	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Tukey HSD	Bryan	Gunther	3.0000*	.57735	.001	1.4597	4.5403
		Marijke	-3.5000*	.57735	.000	-5.0403	-1.9597
	Gunther	Bryan	-3.0000*	.57735	.001	-4.5403	-1.4597
		Marijke	-6.5000*	.57735	.000	-8.0403	-4.9597
	Marijke	Bryan	3.5000*	.57735	.000	1.9597	5.0403
		Gunther	6.5000*	.57735	.000	4.9597	8.0403
Scheffe	Bryan	Gunther	3.0000*	.57735	.001	1.3906	4.6094
		Marijke	-3.5000*	.57735	.000	-5.1094	-1.8906
	Gunther	Bryan	-3.0000*	.57735	.001	-4.6094	-1.3906
		Marijke	-6.5000*	.57735	.000	-8.1094	-4.8906
	Marijke	Bryan	3.5000*	.57735	.000	1.8906	5.1094
		Gunther	6.5000*	.57735	.000	4.8906	8.1094
Bonferroni	Bryan	Gunther	3.0000*	.57735	.001	1.3953	4.6047
		Marijke	-3.5000*	.57735	.000	-5.1047	-1.8953
	Gunther	Bryan	-3.0000*	.57735	.001	-4.6047	-1.3953
		Marijke	-6.5000*	.57735	.000	-8.1047	-4.8953
	Marijke	Bryan	3.5000*	.57735	.000	1.8953	5.1047
		Gunther	6.5000*	.57735	.000	4.8953	8.1047

JASP Multiple Comparisons from ANOVA command.

Post Hoc Comparisons - teacher

		Mean Difference	95% CI for Mean Difference Lower	Upper	SE	t	p_{tukey}	$p_{scheffe}$	p_{bonf}
Bryan	Gunther	3.000	1.436	4.564	0.598	5.020	< .001	< .001	< .001
	Marijke	−3.500	−5.064	−1.936	0.598	−5.857	< .001	< .001	< .001
Gunther	Marijke	−6.500	−8.064	−4.936	0.598	−10.877	< .001	< .001	< .001

Note. P-value and confidence intervals adjusted for comparing a family of 3 estimates (confidence intervals corrected using the tukey method).
Note. Results are averaged over the levels of: sex

7. APA Style Results

Table 4: Descriptive Statistics and Correlations Among Math Scores, Student Sex, and Teachers

Variable	Correlations			
	Math Scores	Male	Bryan	Marijke
Math Scores	---			
Male	.67*	---		
Bryan	-.03	.00	---	
Marijke	.63*	.00	-.50*	---
Mean	78.17	0.50	0.33	0.33
SD	3.87	0.51	0.49	0.49

Note: Male (male = 1, female = 0), Bryan (=1, others = 0) and Marijke (=1, others = 0) are dummy variables; n = 18.
*p < .05.

(Optional: provide table of math score means by teacher)

Table 5: Regression of Math Scores on Student Sex and Teachers

Variable	b	se	ΔR^2	95% CI	F	t
Male	5.00	0.49	.44	3.95, 6.05		10.25*
Teacher			.50		59.27*	
Bryan	3.00	0.60		1.72, 4.28		5.02*
Marijke	6.50	0.60		5.22, 7.78		10.88*
Intercept	72.50	0.49		71.45, 73.55		148.58*

Note: R^2 = .94, adj. R^2 = .93, $F_{3,14}$ = 74.51*, MSE = 1.071, n = 18. ΔR^2 represents the semi-partial correlation or the increment in R^2 due to adding the respective variable. Male (male = 1, female = 0), Bryan (=1, others = 0) and Marijke (=1, others = 0) are dummy variables.
*p < .05.

Table 6: Comparisons of Adjusted Mean Math Scores Among Teachers

Comparison	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
Bryan vs. Gunther	3.00*	0.598	1.38, 4.62
Marijke vs. Gunther	6.50*	0.598	4.88, 8.12
Bryan vs. Marijke	-3.50*	0.598	-5.12, -1.88

*p<.05, where p-values are adjusted using the Bonferroni method.

Regression results show that both student sex and teachers are statistically related to students' math scores at the .05 level of significance. Males score about 5 points higher than females, and students in Marijke's class tend to score higher than students in either of Bryan's or Gunther's class. Students in Gunther's class score lower than in either Bryan's or Marijke's class. Note that all teacher comparisons are statistically different.

8. Exercises

(1) According to the leadership literature, there are a number of different leadership styles. Listed below are scores obtained from an instrument designed to measure a particular leadership style, which will be referred to as style X. Of interest is whether X differs by school district type in terms of urbanity, and by sex. A stratified random sample of school principals were selected from three district types (mostly urban, mostly suburban, and mostly rural).

The scores on style X range from 100 to 0. The closely the score to 100, the more the respondent conforms to style X, while the closer the score to 0, the less the respondent conforms to style X.

Is there any evidence that X differs among the three district types, or by sex?

Sex	District Type	Style X
m	urban	85
m	urban	98
m	urban	75
f	urban	63
m	urban	91
f	urban	49
f	urban	62
f	suburban	49
f	suburban	48
m	suburban	56
m	suburban	78
f	suburban	35
m	suburban	50
m	rural	33
m	rural	95
f	rural	26
f	rural	11
f	rural	33
m	rural	25
m	rural	65

(2) A researcher is interested in learning whether frequency of reading at home to elementary-aged children produces differential effects on reading achievement. After obtaining information from a randomly selected sample of parents about this behavior, the following classifications and standardized achievement scores were recorded. (Note: frequency classifications as follows: a = less than once per month, b = once to three times per month, c = more than three times per month.) In addition to reading frequency, information regarding the family's status concerning whether or not the family's child receives either free or reduced lunch is recorded as a proxy for SES.

SES	Freq. of Reading	Achievement
fr	a	48
fr	a	37
no	a	47
no	a	65
no	b	57
fr	b	39
fr	b	49
no	b	45
no	c	61
no	c	55
fr	c	51
fr	c	30

Note. FR indicates free or reduced lunch received, NO indicates otherwise.

Is frequency of reading at home related to student reading achievement once SES is taken into account?

(3) An administrator wishes to know whether student behavioral problems can be linked to student performance. If students were suspended or reprimanded more than once, they are classified as having behavioral problems. In addition, each student's SES is known, and should be taken into account. The administrator randomly selects 13 students and collects the appropriate data.

Student	GPA	Student SES	Behavioral Problems
Bill	3.33	h	n
Bob	1.79	l	y
Stewart	2.21	m	n
Linda	3.54	h	y
Lisa	2.89	m	n
Ann	2.54	m	n
Fred	2.66	h	y
Carter	1.10	l	y
Bill	3.10	h	n
Sue	2.10	l	y
Kara	2.07	l	y
Loser	2.31	m	n
Kathy	3.67	h	n

9. Exercise Answers

(1) Results for leadership style analysis.

Table 1a
Descriptive Statistics for Leadership Style, District Type, and Sex

Variable	Correlations			
	Style	Urban	Suburban	Male
Style	---			
Urban	.55*	---		
Suburban	-.10	-.48*	---	
Male	.54*	.03	-.07	---
Mean	56.35	.350	.300	.550
SD	25.07	.489	.470	.510

Note: Male is a dummy variable (male = 1, female = 0), as are Urban (1, 0 = other) and Suburban (1, 0 = other); n = 20.

Table 1b
Regression of Style on Sex and District Type

Variable	b	se	ΔR^2	95%CI	F	t
Male	26.29	7.53	.29	10.32, 42.26		3.49*
District Type			.33		7.14*	
Urban	33.57*	8.94		14.62, 52.52		3.76*
Suburban	13.40	9.32		-6.36, 33.16		1.44
Intercept	26.12	7.65		9.91, 42.33		3.42*

Note: R^2 = .625, adj. R^2 = .555, $F_{3,16}$ = 8.90, MSE = 279.70, n = 20. ΔR^2 represents the semi-partial correlation or the increment in R^2 due to adding the respective variable. Male is a dummy variable (male = 1, female = 0), as are Urban (1, 0 = other) and Suburban (1, 0 = other).
*p < .05.

Table 1c
Comparisons of Style Scores Among Urban, Suburban, and Rural Principals

Contrast	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
Urban vs. Rural	33.57*	8.94	9.74, 57.40
Suburban vs. Rural	13.40	9.32	-11.44, 38.24
Urban vs. Suburban	20.17	9.32	-4.67, 45.01

*p<.05, where p-values are adjusted using the Bonferroni method.

[Note, Bonferroni CI taken from Excel Spreadsheet is incorrect so must calculate CI using tabled values for Bonferroni comparisons. Use male = .55 in regression equation to obtain estimated means for each district.]

Both sex and district type are statistically related to leadership style. Once district type is taken into account, males average about 26 points higher than females. Among the three district types considered, principals in urban settings have a statistically higher score on style than do principals in rural districts, but not statistically higher than principals in suburban districts.

(2) Results for reading frequency.

Table 2a
Descriptive Statistics for Achievement, SES, and Reading Frequency

Variable	Correlations			
	Achievement	B	C	SES
Achievement	---			
B = 1 to 3 per month	-.09	---		
C = more than 3 per month	.04	-.50	---	
SES	-.65*	.00	.00	---
Mean	48.66	.333	.333	.500
SD	10.129	.492	.492	.522

Note: SES is a dummy variable (free/reduced lunch = 1, otherwise = 0), as are B (1, 0 = other) and C (1, 0 = other); n = 12.

Table 2b
Regression of Achievement on Reading Frequency and SES

Variable	b	se	ΔR^2	95%CI	F	t
SES	-12.66	5.16	.426	-24.57, -0.77		-2.45*
Reading Freq.			.007		0.05	
B	-1.75	6.32		-16.32, 12.82		-0.28
C	-0.00	6.32		-14.57, 14.57		0.00
Intercept	55.58	5.16		43.68, 67.48		10.77*

Note: R^2 = .43, adj. R^2 = .22, $F_{3,8}$ = 2.04, MSE = 79.89, n = 12. ΔR^2 represents the semi-partial correlation or the increment in R^2 due to adding the respective variable. SES is a dummy variable (free/reduced lunch = 1, otherwise = 0), as are B (1, 0 = other) and C (1, 0 = other).
*p < .05.

Table 2c
Comparisons of Achievement among Reading Frequency

Contrast	Estimated Mean Difference	Standard Error of Difference	.95CI
B vs. A	-1.75	6.32	-16.32, 12.82
C vs. A	-0.00	6.32	-14.57, 14.57
B vs. C	-1.75	6.32	-16.32, 12.82

*p < .05.

[Note – the above comparison represents the unadjusted comparisons (no Bonferroni corrections); these numbers obtained from regression output. Bonferroni adjusted comparisons reported below in 2d.]

Table 2d
Comparisons of Adjusted Mean Reading Achievement Scores

Comparison	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
A vs. c	0.00	6.32	-18.99, 18.99
B vs. c	-1.75	6.32	-20.74, 17.24
A vs b	1.75	6.32	-17.24, 20.74

*p<.05, where p-values are adjusted using the Bonferroni method.

Only SES was statistically related to achievement scores, with those receiving free for reduced lunch scoring about 12 to 13 points lower than those not receiving free/reduced lunch, on average. There were no statistical differences observed among the three levels of reading frequency.

Bonferroni and Scheffe adjusted confidence intervals are reported below.

```
. regr achievement i.read_freq_num i.ses_num
. pwcompare read_freq_num, bonf
```

Pairwise comparisons of marginal linear predictions

```
---------------------------------------------------------------
              |                                 Bonferroni
              |   Contrast   Std. Err.     [95% Conf. Interval]
--------------+------------------------------------------------
read_freq_num |
      2 vs 1  |      -1.75   6.320436     -20.81093    17.31093
      3 vs 1  |   7.07e-15   6.320436     -19.06093    19.06093
      3 vs 2  |       1.75   6.320436     -17.31093    20.81093
---------------------------------------------------------------

. pwcompare read_freq_num, sch
---------------------------------------------------------------
              |                                  Scheffe
              |   Contrast   Std. Err.     [95% Conf. Interval]
--------------+------------------------------------------------
read_freq_num |
      2 vs 1  |      -1.75   6.320436     -20.62467    17.12467
      3 vs 1  |   7.07e-15   6.320436     -18.87467    18.87467
      3 vs 2  |       1.75   6.320436     -17.12467    20.62467
---------------------------------------------------------------
```

(3) Results for GPA analysis.

Table 3a
Descriptive Statistics for GPA, SES, and Behavioral Problems

Variable	Correlations			
	GPA	High	Mid	Behavior
GPA	---			
High SES	.78*	---		
Mid. SES	-.07	-.53	---	
Behavior	-.46	-.10	-.62*	---
Mean	2.56	0.39	0.31	0.46
SD	0.74	0.51	0.48	0.52

Note: High (1, 0 = otherwise) and Mid. SES (1, 0 = otherwise) are dummy variables, as is behavior (1 for problems, 0 = otherwise); n = 13.

Table 3b
Regression of GPA on Behavioral Problems and SES

Variable	b	se	ΔR^2	95%CI	F	t
Behavioral Prob.	-.27	.37	.01	-1.10, 0.57		-0.72
SES			.56		11.31*	
High	1.34*	.35		0.54, 2.13		3.81*
Mid	.46	.47		-0.60, 1.51		0.98
Intercept	2.03	.42		1.08, 2.98		4.83*

Note: R^2 = .78, adj. R^2 = .70, $F_{3,9}$ = 10.35*, MSE = 0.164, n = 13. ΔR^2 represents the semi-partial correlation or the increment in R^2 due to adding the respective variable.
*p < .05.

Table 3c
Comparisons of Achievement among Reading Frequency

Contrast	Estimated Mean Difference	Standard Error of Difference	95% CI of Mean Difference
High vs. Low	1.34*	0.35	0.54, 2.21
Mid vs. Low	0.46	0.47	-.60, 1.51
High vs. Mid	0.88	0.31	.18, 1.58

*p < .05.

Only SES was statistically related to GPA, with those in the high SES group showing statistically higher GPAs than either the middle or low SES groups. There was no statistical difference between the middle and low SES groups, nor was behavioral problem associated with GPA.

[Table 3c above are the unadjusted comparisons, Table 3d below contains the Bonferroni adjusted comparisons using the estimated means with behavioral problems mean used as 0.46 to obtained predicted means for each of the three SES groups.]

[Again note that the Excel spreadsheet se are too small and erroneous, so use tabled Bonferroni critical t and calculate CI using regression se.]

Table 3c
Comparisons of Achievement among Reading Frequency

Contrast	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
High vs. Low	1.34*	0.35	0.31, 2.36
Mid vs. Low	0.46	0.47	-0.91, 1.83
High vs. Mid	0.88	0.31	-0.03, 1.79

*p<.05, where p-values are adjusted using the Bonferroni method.

[Bonferroni critical t = 2.923 (3 comparions, 9 df)]

Scheffé confidence intervals are reported below.

Table 3c
Comparisons of Achievement among Reading Frequency

Contrast	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
High vs. Low	1.34*	0.35	0.31, 2.36
Mid vs. Low	0.46	0.47	-0.91, 1.82
High vs. Mid	0.88	0.31	-0.02, 1.78

*p<.05, where p-values are adjusted using the Scheffé method.