# Simple Linear Regression: One Dichotomous IV

## 1. Purpose

As noted before regression is used both to explain and predict variation in DVs, and adding to the equation categorical variables extends regression flexibility and enables one to perform group contrasts in a way that is mathematically identical to ANOVA. Simple linear regression with one qualitative IV variable is essentially identical to linear regression with quantitative variables. The primary difference between the two is how one interprets the regression coefficients.

## 2. Regression Equations

### Population

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \qquad (1)$$

where

$Y_i$ = the $i^{th}$ student's score,
$\beta_1$ = population regression coefficient expressing the relationship between X and Y,
$\beta_0$ = population intercept for the equation, and
$\varepsilon_i$ = random error term.

### Population Prediction Equation

$$Y' = \beta_0 + \beta_1 X_i$$

Where Y' is the predicted value of the DV in the population. Note absence of $\varepsilon$; since means are predicted based upon the equation, individual score deviations from the prediction are not included.

### Sample

$$Y_i = b_0 + b_1 X_i + e_i, \qquad (2)$$

where

$b_0$ is the sample intercept, $b_1$ is the sample regression coefficient, and e is the sample residual term in the model.

Note: Error denotes population deviations and residual denotes sample deviations.

### Sample Prediction Equation

$$Y' = b_0 + b_1 X_i$$

Note that the above equations are exactly the same as found when the IV is quantitative (refer back to notes on regression with one quantitative predictor).

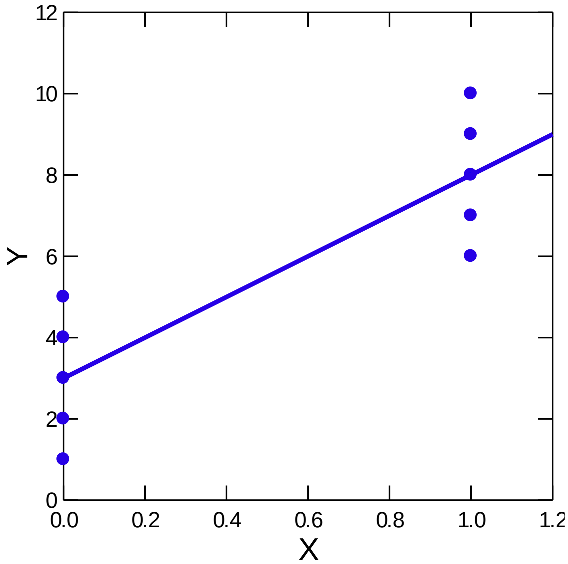
## 2. Fictional Data for Group Comparison

Table 1: Fictional Data by Sex (X)

| Y | X | X = Sex | Mean |
|---|---|---|---|
| 1 | 0 | M | |
| 2 | 0 | M | Male |
| 3 | 0 | M | m = 3 |
| 4 | 0 | M | |
| 5 | 0 | M | |
| 6 | 1 | F | |
| 7 | 1 | F | Female |
| 8 | 1 | F | m = 8 |
| 9 | 1 | F | |
| 10 | 1 | F | |

Note there are two groups X = 0 and X = 1 (i.e., X = 0 = Male, X = 1 = Female).

**Slope for Fictional Data**




JASP Regression Results for Fictional Data

Coefficients

| Model | | Unstandardized | Standard Error | Standardized | t | p |
|---|---|---|---|---|---|---|
| $H_0$ | (Intercept) | 5.500 | 0.957 | | 5.745 | < .001 |
| $H_1$ | (Intercept) | 3.000 | 0.707 | | 4.243 | 0.003 |
| | Sex-dummy | 5.000 | 1.000 | 0.870 | 5.000 | 0.001 |

(a) Given this regression equation, $Y' = b_0 + b_1X_i$, what would be the value of $b_0$?

Since bo is the mean for the comparison group (dummy = 0, m = 3) it will equal 3

(b) What would be the value of $b_1$?

Since b1 is the difference between the dummy group 1 (female) and group 0 (males) it will be 8 – 3 = 5.

(c) Find the coefficient estimates for the fictional data in SPSS or JASP.

Enter data and run regression! See above JASP results.

$Y_i = b_0 + b_1X_i + e_i$,
$Y_i = 3.0 + 5.0(X_i) + e_i$,

(d) What is the literal interpretation for $b_0$?

It is the predicted value of Y for the reference group when X = 0.

(e) What is the literal interpretation for $b_1$?

It is the mean difference in Y between the two modeled groups, the reference group (X = 0) and the comparison group (X = 1).

**3. Dummy Variable Coding**

One method for representing categorical IV in regression is dummy coding. One group is coded with 0, the other with 1. The variable X in Table 1 above represents dummy coding for sex. A dummy variable in regression represents the mean difference between the groups coded with 0 and 1. In the current example, the regression equation is:

$Y_i = b_0 + b_1X_i + e_i$,
$Y_i = 3.0 + 5.0(X_i) + e_i$,

Males are coded as X = 0, females coded as X = 1, so:

Predicted Y' mean for males = $Y_i = b_0 + b_1X_i$
Predicted Y' mean for males = $Y_i = b_0 + b_1 \times 0$
Predicted Y' mean for males = $Y_i = b_0$

Predicted Y' mean for males = $Y_i = 3.0 + 5.0(X_i)$
Predicted Y' mean for males = $Y_i = 3.0 + 5.0(0)$
Predicted Y' mean for males = $Y_i = 3.0$

and the predicted Y' mean for females

Predicted Y' mean for females = $Y_i = b_0 + b_1X_i$
Predicted Y' mean for females = $Y_i = b_0 + b_1 \times 1$
Predicted Y' mean for females = $Y_i = b_0 + b_1$

Predicted Y' mean for females = $Y_i = 3.0 + 5.0(X_i)$
Predicted Y' mean for females = $Y_i = 3.0 + 5.0(1)$
Predicted Y' mean for females = $Y_i = 3.0 + 5.0$
Predicted Y' mean for females = $Y_i = 8.0$

Given the above, note the following:

$b_0$ = predicted Y' mean for group coded with dummy = 0;
$b_0 + b_1$ = predicted Y' mean for group coded with dummy = 1;
$b_1$ = predicted mean difference in Y between the two groups.

**4. Fictional Data Example #2**

Using the data below, create a dummy variable to represent student classification level.

Table 2: Fictional Data of Attitude Toward Recreation Activity Center and Student Classification Level

| Attitude toward RAC at GSU | Student Level | | Dummy Variable |
|---|---|---|---|
| 2 | Graduate | 1 | 0 |
| 3 | Graduate | 1 | 0 |
| 5 | Graduate | 1 | 0 |
| 1 | Graduate | 1 | 0 |
| 2 | Graduate | 1 | 0 |
| 4 | Undergraduate | 0 | 1 |
| 5 | Undergraduate | 0 | 1 |
| 3 | Undergraduate | 0 | 1 |
| 5 | Undergraduate | 0 | 1 |
| 4 | Undergraduate | 0 | 1 |

Note: Attitude toward RAC scoring: 1 = very unhappy, 5 = very happy

JASP Regression Model for RAC Attitudes and Student Level

Coefficients ▼

| Model | | Unstandardized | Standard Error | Standardized[a] | t | p | 95% CI Lower | 95% CI Upper |
|---|---|---|---|---|---|---|---|---|
| $H_0$ | (Intercept) | 3.400 | 0.452 | | 7.520 | < .001 | 2.377 | 4.423 |
| $H_1$ | (Intercept) | 2.600 | 0.548 | | 4.747 | 0.001 | 1.337 | 3.863 |
| | StudentLevel (Undergraduate) | 1.600 | 0.775 | | 2.066 | 0.073 | −0.186 | 3.386 |

[a] Standardized coefficients can only be computed for continuous predictors.

Answer the following questions.

(a) What is the coefficient estimates for this model, that is, what are the values for $b_0$ and $b_1$?

$Y_i = b_0 + b_1X_i + e_i$,
$Y_i = 2.6 + 1.6(X_i) + e_i$,

(b) What is the literal interpretation for $b_0$?

Predicted value of RAC attitude for graduate students, 2.60

(c) What is the literal interpretation for $b_1$?

The mean difference between undergraduate and graduate students, 1.6

(d) What is the predicted mean level of attitude for graduate students?

$Y' = 2.6 + 1.6(0) = 2.6 + 0 = 2.6$
$b_0 = 2.6$

(e) What is the predicted mean level of attitude for undergraduate students?

$Y' = 2.6 + 1.6(X_i)$
$Y' = 2.6 + 1.6(1) = 2.6 + 1.6 = 4.2$

**5. Inferential Procedures for Regression Coefficient**

Hypothesis testing is performed in the same way as discussed previously with regression.

**5a. t-ratios**

$b_1/se$ = t-ratio

and

$H_0$: $\beta_1 = 0.00$
$H_1$: $\beta_1 \neq 0.00$

- Recall that $\beta_1$ is the mean difference between the two groups.
- If $H_0$ is not rejected, then one may conclude that the groups do not differ statistically.
- If, however, $H_0$ is rejected, then one may conclude that the mean difference between the groups is statistically significant.

**5b. Confidence Intervals**

If 0.00 lies within the confidence interval, fail to reject Ho.
If 0.00 does not lie within the confidence interval, reject Ho.

**Dummy Variable for Two Groups: t-test and ANOVA Linkage**

Note that $H_0$ listed above is identical to the null hypothesis for the two-independent samples t-test:

$H_0$: $\mu_1 = \mu_2$

or

$H_0$: $\mu_1 - \mu_2 = 0.00$,

that is, the difference between the two groups is 0.00.

Also note that the null hypothesis for ANOVA with only two groups is identical to the two-independent samples t-test and regression with a dummy IV. The null in ANOVA for two groups is:

$H_0$: $\mu_1 = \mu_2$

As these hypotheses reveal, regression with a dummy variable is identical to the two-independent samples t-test and to ANOVA with two groups.

(a) Show result similarity with the above fictional data using t-test and ANOVA.
(b) Note when $df_1 = 1$, then $F = t^2$

JASP t-test for RAC Attitude and Student Level

| Independent Samples T-Test | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | 95% CI for Mean Difference | |
| | t | df | p | Mean Difference | SE Difference | Lower | Upper |
| RAC-attitude | −2.066 | 8 | 0.073 | −1.600 | 0.775 | −3.386 | 0.186 |

*Note.* Student's t-test.

Regression and t-test results are the same: mean difference, standard error, t-ratio, p-value, and CI except that signs are reversed since order of group mean comparisons were reversed (grade mean – undergrad mean rather than undergrade mean – grad mean). Below are t-test results with group comparisons reversed to match regression.

JASP t-test for RAC Attitude and Student Level with Order of Means Reversed

| Independent Samples T-Test | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | 95% CI for Mean Difference | |
| | t | df | p | Mean Difference | SE Difference | Lower | Upper |
| RAC-attitude | 2.066 | 8 | 0.073 | 1.600 | 0.775 | −0.186 | 3.386 |

*Note.* Student's t-test.

JASP ANOVA RAC Attitude Results

ANOVA - RAC-attitude ▼

| Cases | Sum of Squares | df | Mean Square | F | p |
|---|---|---|---|---|---|
| StudentLevel | 6.400 | 1 | 6.400 | 4.267 | 0.073 |
| Residuals | 12.000 | 8 | 1.500 | | |

*Note.* Type III Sum of Squares

## Descriptives

Descriptives - RAC-attitude

| StudentLevel | N | Mean | SD | SE | Coefficient of variation |
|---|---|---|---|---|---|
| Graduate | 5 | 2.600 | 1.517 | 0.678 | 0.583 |
| Undergraduate | 5 | 4.200 | 0.837 | 0.374 | 0.199 |

## Post Hoc Tests

### Standard

Post Hoc Comparisons - StudentLevel

| | | Mean Difference | SE | t | $p_{tukey}$ |
|---|---|---|---|---|---|
| Undergraduate | Graduate | 1.600 | 0.775 | 2.066 | 0.073 |

The ANOVA results above also match regression with the same F ratio (4.267) and p-value (0.073), sums of squares and mean squares, and degrees of freedom.

**6. Confidence Intervals (CI)**

CI represents an upper and lower bound to the point estimate of the regression coefficient. Thus, CIs indicate the precision of the particular estimate. The CI for b is:

b $\pm$ critical $t_{(\alpha/2,df)} \times SE_b$

where t is the critical t-value obtained from a table of t values representing a two-tailed alpha ($\alpha$) level (such as .05) with degrees of freedom equal to n-k-1, and $SE_b$ is the standard error for the regression coefficient.

Also, as before, if 0.00 lies within the CI, one will fail to reject $H_0$.

**7. Model Fit**

As before, model fit remains the same: $R^2$, adjusted $R^2$, SEE (standard error of estimate; the standard deviation of the residuals), and MSE (mean squared estimate, variance of the residuals). For example:

$R^2$ = Squared Pearson r between DV and Predicted DV

adj. $R^2 = 1 - \frac{MSE}{VAR(Y)}$ = *calculate for current example*

Similarly, residuals are calculated in the same way:

$e = Y - Y'$

adj. $R^2$ = Squared Pearson r between DV and Predicted DV

(a) Calculate e for first observation in sample data.
(b) Calculate e for last observation in sample data.

**8. Overall Model Fit and Statistical Inference**

To test the regression model as a whole—to learn whether any of the predictors are related to Y, or to learn whether the combination of predictors predict more variation in Y than one would expect by chance—one may test whether $R^2$ differs from 0.00:

$H_0$: $R^2 = 0$

or

$H_0$: $\beta_j = 0.00$ (all regression slopes equal 0.00)

As before, the overall F test is used to test $H_0$. F is calculated, like ANOVA, using any of the following formulae:

$$F = \frac{SSR/df_1}{SSE/df_2} = \frac{SSR/k}{SEE/(n-k-1)} = \frac{MS_R}{MSE}$$

where;

| | | |
|---|---|---|
| SSR | = | regression sums of squares; |
| SSE | = | residual sums of squares; |
| $df_1$ | = | regression degrees of freedom; |
| $df_2$ | = | residual degrees of freedom; |

k = number of independent variables (vectors) in the model;
n = sample size (or number of observations in sample);
$MS_R$ = mean square (same as ANOVA) due to regression (e.g., between);
MSE = mean square error (same as ANOVA mean square within).

As before there are two sources for degrees of freedom:

$df_1 = k$ (number of predictors in regression equation)

and

$df_2 = n - k - 1$

Illustrate calculation of F ratio with current data, find df and critical F – do we reject $H_0$?

JASP regression ANOVA results

ANOVA

| Model | | Sum of Squares | df | Mean Square | F | p |
|---|---|---|---|---|---|---|
| $H_1$ | Regression | 6.400 | 1 | 6.400 | 4.267 | 0.073 |
| | Residual | 12.000 | 8 | 1.500 | | |
| | Total | 18.400 | 9 | | | |

*Note*. The intercept model is omitted, as no meaningful information can be shown.

**9. Reporting Regression Results**

Below is an example of APA styled reporting of results using the first fictional data given in Table 1:

Table 3: Descriptive Statistics and Correlations between RAC Attitude and Student Level

| Variable | Correlations | | |
|---|---|---|---|
| | RAC Attitude | | Undergrad. Indicator |
| RAC Attitude | --- | | |
| Undergraduates | .59 | | --- |
| Mean | 3.40 | | 0.50 |
| SD | 1.43 | | 0.57 |
| n | 10 | | 10 |
| RAC Attitude | M | SD | n |
| Undergraduates | 4.20 | 0.837 | 5 |
| Graduates | 2.60 | 1.517 | 5 |

Note. n = 10. Undergraduate indicator coded 1 = undergraduate students and 0 = graduate students.
* $p < .05$

Table 4: Summary of Regression of RAC Attitude on Student Level

| Variable | b | se b | 95%CI | t |
|---|---|---|---|---|
| Undergraduate | 1.60 | 0.775 | -0.19, 3.87 | 2.07 |
| Intercept | 2.60 | 0.548 | 1.34, 3.86 | 4.74* |

Note. $R^2$ = .35, adj. $R^2$ = .27, F = 4.27, MSE = 1.50, df = 1,8, n = 10; Undergraduate coded 0 = graduates and 1 = undergraduates.
*$p < .05$.

Regression results show that there is not a significant difference, at the .05 level, in RAC attitude means between undergraduate and graduate students. This finding suggests the 1.60 point mean difference between the two groups is possibly due to chance differences (, although the small sample size of n = 10 may also impact the power of this test).

The bit in parentheses can be added when sample sizes are small to acknowledge the effect of small sample sizes on the power of a test to detect differences or relations.

Note: The above regression table does not contain standardized coefficient estimates or estimates of $\Delta R^2$ estimates. For simple regression with one qualitative variable, neither of these estimates are applicable.

**10. Exercises**

(1) A teacher is convinced that frequency of testing within her classroom increases student achievement. She runs an experiment for several years in her algebra class. In some classes, students are exposed to a test every week. In other classes, students are tested three times during the quarter. Is there evidence that testing frequency is related to average achievement?

| Quarter | Testing Frequency During Quarter | Overall Class Achievement on Final Exam |
|---|---|---|
| Fall 1991 | 3 times | 85.5 |
| Winter 1992 | 3 times | 86.5 |
| Spring 1992 | 3 times | 88.9 |
| Summer 1992 | weekly | 89.1 |
| Fall 1992 | 3 times | 87.2 |
| Winter 1993 | weekly | 90.5 |
| Spring 1993 | weekly | 89.8 |
| Summer 1993 | weekly | 92.5 |
| Fall 1994 | weekly | 89.3 |
| Winter 1994 | 3 times | 90.1 |

(2) Two classes of educational research were taught with two different methods of instruction, teacher guided (TG) and self paced (SP). Which had the better student achievement at the end of the quarter?

TG scores: 95, 93, 87, 88, 82, 92
SP scores: 78, 89, 83, 90, 78, 86

(3) A researcher wishes to know whether home study is related to general achievement amongst high school students. Students were surveyed, and all students who indicated that they routinely studied were coded 1, others were coded 0.

| Student | High School GPA | Regularly Study at Home |
|---|---|---|
| Bill | 3.33 | 1 |
| Bob | 1.79 | 0 |
| Stewart | 2.21 | 1 |
| Linda | 3.54 | 1 |
| Lisa | 2.89 | 0 |
| Ann | 2.54 | 1 |
| Fred | 2.66 | 0 |
| Carter | 1.10 | 0 |
| Kathy | 3.67 | 1 |

(4) Determine whether a statistical difference exists between men and women in weight:
Men: 156, 158, 175, 203, 252, 195
Women: 149, 119, 168, 123, 155, 126

**11. Answers to Exercises**

(1)

*Table 1a: Descriptive Statistics and Correlations between Testing Frequency and Algebra Achievement*

| Variable | Correlations | |
|---|---|---|
| | Algebra Final | Testing Dummy |
| Algebra Final | --- | |
| Testing Dummy | .67* | --- |
| Mean | 88.94 | 0.50 |
| SD | 2.06 | 0.53 |

Note. n = 10; Testing Dummy 1 = weekly tests, 0 = three tests per semester.
* $p < .05$

*Table 1b: Regression of Algebra Achievement on Testing Frequency*

| Variable | b | se | 95%CI | t |
|---|---|---|---|---|
| Testing Dummy | 2.60 | 1.03 | 0.22, 4.98 | 2.52* |
| Intercept | 87.64 | 0.73 | 85.96, 89.32 | 120.20* |

Note. $R^2$ = .44, adj. $R^2$ = .37, F = 6.36*, df = 1,8; n = 10;
Testing Dummy 1 = weekly tests, 0 = three tests per semester.
*$p < .05$.

Regression results show that testing frequency appears to be related, at the .05 level of significance, to algebra achievement. Students who tested weekly scored about 2.60 points higher in algebra, on average, than did students who were test three times during the semester.

(2)

*Table 2a: Descriptive Statistics and Correlations between Type of Instruction and Educational Research Achievement*

| Variable | Correlations | |
|---|---|---|
| | Ed. Research Scores | Instruction Dummy |
| Ed. Research Scores | --- | |
| Instruction Dummy | .52 | --- |
| Mean | 86.75 | 0.50 |
| SD | 5.58 | 0.52 |

Note. n = 10; Instruction Dummy 1 = teacher guided, 0 = self-paced.
* $p < .05$

*Table 2b: Regression of Type of Instruction on Educational Research Scores*

| Variable | b | se | 95%CI | t |
|---|---|---|---|---|
| Testing Dummy | 5.50 | 2.90 | -0.95, 11.95 | 1.90 |
| Intercept | 84.00 | 2.05 | 79.44, 88.56 | 41.03* |

Note. $R^2$ = .27, adj. $R^2$ = .19, F = 3.61, df = 1,10; n = 12;
Instruction Dummy 1 = teacher guided, 0 = self-paced.
*$p < .05$.

Regression results show that there is not a statistical difference in mean educational research scores between those in teacher guided instruction and those in self-paced instruction. Students appear to perform similarly whether in teacher guided or self-paced instruction.

(3)

*Table 3a: Descriptive Statistics and Correlations between GPA and Home Study*

| Variable | Correlations | |
|---|---|---|
| | GPA | Home Study |
| GPA | --- | |
| Home Study Dummy | .59 | --- |
| Mean | 2.64 | 0.56 |
| SD | 0.84 | 0.53 |

Note. n = 9; Home Study Dummy 1 = regularly studies at home, 0 = does not regularly study at home.
* $p < .05$

*Table 3b: Regression of GPA on Home Study*

| Variable | b | se | 95%CI | t |
|---|---|---|---|---|
| Home Study Dummy | 0.95 | 0.49 | -0.21, 2.10 | 1.94 |
| Intercept | 2.11 | 0.36 | 1.25, 2.97 | 5.80* |

Note. $R^2$ = .35, adj. $R^2$ = .26, F = 3.78, df = 1,7; n = 9;
Home Study Dummy 1 = regularly studies at home, 0 = does not regularly study at home.
*$p < .05$.

Regression results show that there is not a statistical difference in mean GPA between students who regularly study at home and those who do not regularly study at home. GPA appears to be similar for both those who do and do not regularly study at home.

(4)

*Table 4a: Descriptive Statistics and Correlations between Sex and Weight*

| Variable | Correlations | |
|---|---|---|
| | Sex Dummy | Weight |
| Sex Dummy | --- | |
| GPA | .68* | --- |
| Mean | 0.50 | 164.92 |
| SD | 0.52 | 38.03 |

Note. n = 12; Sex Dummy 1 = males, 0 = females.
* $p < .05$

*Table 4b: Regression of Weight on Sex*

| Variable | b | se | 95%CI | t |
|---|---|---|---|---|
| Sex Dummy | 49.83 | 16.79 | 12.42, 87.25 | 2.97* |
| Intercept | 140.00 | 11.87 | 113.54, 166.46 | 11.79* |

Note. $R^2$ = .47, adj. $R^2$ = .42, F = 8.81*, df = 1,10; n = 12;
Sex Dummy 1 = males, 0 = females.
*$p < .05$.

Regression results show that there is a statistically significant difference in mean weight between male and female participants—males, on average, weigh about 50 pounds more than females.