Notes 9c: Two-way ANOVA with Interactions

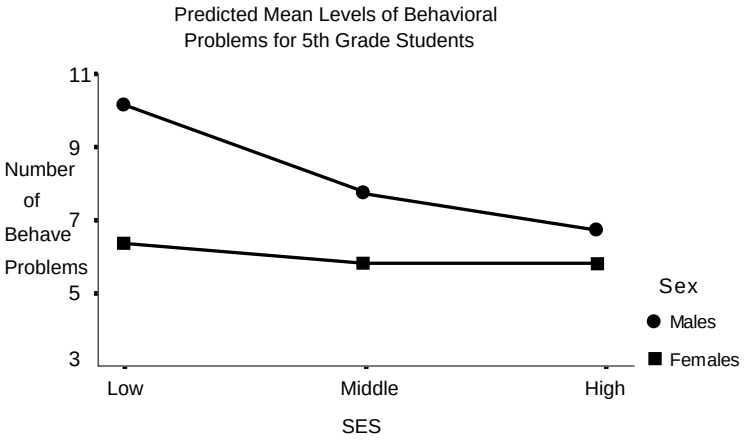
1. What is an Interaction?

An interaction occurs when an independent variable's statistical effects (or differences) upon the dependent variable varies or differ across levels of a second independent variable.

(a) Examples of Interactions

Figure one For example, if one were interested in examining the relationship between SES, sex, and the number of behavioral problems displayed among 5th grade students, one may find a pattern such as the one depicted in Figure 1.

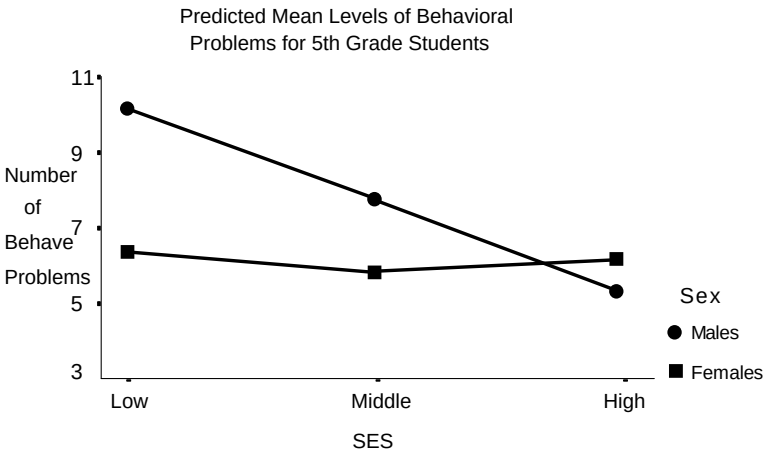
Figure 1: Ordinal Interaction



As illustrated in Figure 1, the difference in behavioral problems between males and females changes across levels of SES; thus, there is an interaction between SES and sex.

Note that an interaction only occurs between independent variables; the dependent variable does not interact with an independent variable. There are two types of interactions, ordinal and dis-ordinal. Figure 1 illustrates an ordinal interaction. An ordinal interaction occurs when one group's predicted means is always greater than another group's predicted means. For example, the predicted male means are always greater than predicted female means, yet the differences between males and females varies by SES, therefore an ordinal interaction results.

Figure 2: Dis-ordinal Interaction

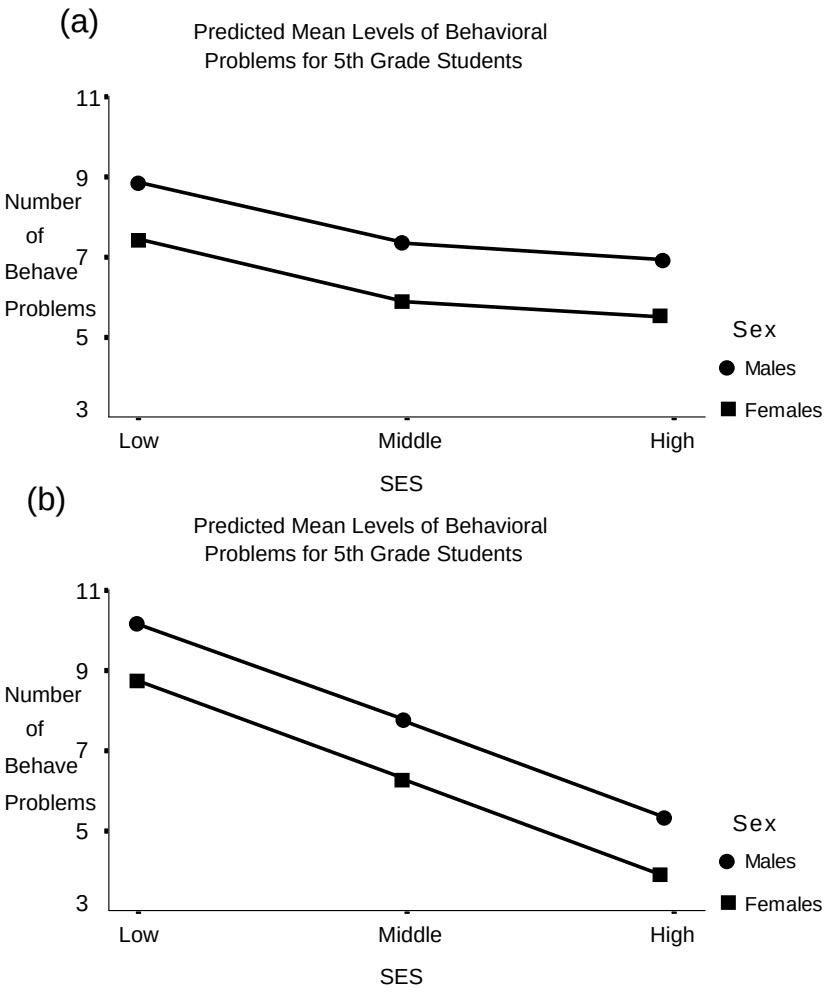


When two or more group means switch or cross, a disordinal interaction occurs. Figure 2 illustrates a disordinal interaction. Note that predicted male means are higher for low and middle levels of SES, but the predicted female mean for high levels of SES is greater than the predicted male mean.

(b) Examples Without Interactions

Figures 3a and 3b, which contain no interactions, are included as a reference. Note that the differences between males and females is constant in both a and b, i.e., the sex difference does not vary by SES level.

Figure 3



2. Two-way ANOVA with Interaction

Sometimes interactions can mask main effects of factors (IVs). Below is a very simple example illustrating the masked effect using achievement as the DV and instruction type and student sex as the IV or factors.

Table 1
Example Date for Two-way ANOVA with Interaction

Achievement	Instruction Type	Sex
74	Co-operative	m
76	Co-operative	m
84	Co-operative	f
86	Co-operative	f
78	Lecture	m
82	Lecture	m
79	Lecture	f
81	Lecture	f
86	Self-paced	m
84	Self-paced	m
76	Self-paced	f
74	Self-paced	f

(a) Data Plotted – What do the plots show?

These data are plotted here:

https://spreadsheets.google.com/ccc?key=0AoKw33oyzB1NdDVTcnk5ZVhOaWp0bGpERHNBQ3hhdEE&hl=en&authkey=COzWuIYD

(b) Table of Means and Marginal Means – What does this show?

Table 2
Means and Marginal Means for Sample

	Coop	Lecture	Self	Marginal Means for Sex
Female	85.00	80.00	75.00	80.00
Male	75.00	80.00	85.00	80.00
Marginal Means for Instruction	80.00	80.00	80.00	

With these marginal means, would we reject the null hypothesis for sex or for instruction?

H_0(sex): $\mu_{males.} = \mu_{females.}$

H_0(instruction): $\mu_{.coop} = \mu_{.lecture} = \mu_{.self}$

As before the dots indicate average across levels of the other factors, that is, hypotheses tests are based upon marginal means.

(c) ANOVA Summary Without Interaction – What do these results indicate?

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.000(a)	3	.000	.000	1.000
Intercept	76800.000	1	76800.000	2818.349	.000
instruction	.000	2	.000	.000	1.000
sex	.000	1	.000	.000	1.000
Error	218.000	8	27.250		
Total	77018.000	12			
Corrected Total	218.000	11			

a R Squared = .000 (Adjusted R Squared = -.375)

1. instruction

instruction	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Co-operative	80.000	2.610	73.981	86.019
Lecture	80.000	2.610	73.981	86.019
Self-paced	80.000	2.610	73.981	86.019

2. sex

sex	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
f	80.000	2.131	75.086	84.914
m	80.000	2.131	75.086	84.914

In summary, what is the possible result of the ANOVA hypothesis tests if only marginal means are examined without regard to interactions?

(d) ANOVA Summary With Interaction – What do these results indicate?

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	200.000(a)	5	40.000	13.333	.003
Intercept	76800.000	1	76800.000	25600.000	.000
instruction	.000	2	.000	.000	1.000
sex	.000	1	.000	.000	1.000
instruction * sex	200.000	2	100.000	33.333	.001
Error	18.000	6	3.000		
Total	77018.000	12			
Corrected Total	218.000	11			

a R Squared = .917 (Adjusted R Squared = .849)

3. Two-way ANOVA Model with Interaction

Two-way ANOVA with interaction is simply an ANOVA with two qualitative independent variables and the interaction between them.

(a) Regression Model Without Interaction

The multiple regression model for a two-way ANOVA without interaction looks like this:

$Y' = b_0 + b_1 coop + b_2 self + b_3 female$

where "coop," "self," and "female" are dummy variables and the omitted categories (reference group) are males in the lecture treatment.

(b) Creating Interactions in Regression

To form interactions one multiples the IVs and includes those product terms in the regression model. For example, consider two predictors X and Z. As a general example, the interaction between X and Z is found by including in the regression model the following product:

Interaction (XZ) = X * Z.

When one or more of the independent variables is categorical, each dummy representing the variable must be multiplied. For the current example, there will be two multiplicative terms to include in the regression model:

coop×female = coop × female

self×female = self × female

(c) Regression Model With Interaction

Entering these terms into the sample regression model results in the following equation:

$Y' = b_0 + b_1 coop + b_2 self + b_3 female + b_4 coop \times female + b_5 self \times female$

(d) ANOVA with interaction

In most software one does not have to produce the underlying linear model as is done in regression above. Instead, one simply specifies the model and the software creates the linear equation. For ANOVA the model would appear as follows:

Y' = Intercept + Instruction + Sex + I×S

where "I" represents instruction and "S" represents sex.

4. Hypotheses

The hypotheses remain essentially unchanged from previously with ANOVA except comparisons now must be specified as "main effect" or "simple main effect."

(a) Main Effect Tests

This is the same hypothesis test of group comparisons that have been discussed previously in ANOVA, i.e.:

Instruction

H_0: $\mu_{1.} = \mu_{2.} = \mu_{3.}$ (taking mean across both males and females for each category of instruction)
H_a: not all of the instructional treatments means are equal

Sex

H_0: $\mu_{1.} = \mu_{2.}$ (taking mean across all levels of instruction)
H_a: $\mu_{1.} \neq \mu_{2.}$.

(b) Interaction Test

Interaction between the two independent variables

H_0: $\alpha\beta = 0$
H_a: $\alpha\beta \neq 0$

where $\alpha\beta$ represent the interaction product of multiplying two factors such as sex and instruction.

(c) Simple Main Effects

If the interaction is statistically significant, then focus will be upon not Main Effects tests, but upon Simple Main Effects tests. A simple main effect just means that one makes comparisons, or hypotheses tests, for one variable by each level (or category) of the second variable if both variables are categorical.

Simple main effects hypothesis testing can also occur where the IVs are quantitative. The situation where one IV is quantitative and the second is qualitative will be discuss in ANCOVA. In the situation where both IVs are quantitative will be covered in regression.

As the example data discussed above "**2. Two-way ANOVA with Interaction**" shows, if one examines the marginal means the interpretation of results can be misleading if an interaction is present. This may not be true in some types of interactions (such as a weak ordinal interaction), but one should examine the simple main effects whenever an interaction is statistically significant.

Table 3
Means and Marginal Means for Sample

	Coop	Lecture	Self	Marginal Means for Sex
Female	85.00	80.00	75.00	80.00
Male	75.00	80.00	85.00	80.00
Marginal Means for Instruction	80.00	80.00	80.00	

(c.1) Sex Simple Main Effects

Requires three separate t-tests, one for each instructional condition:

Cooperative Learning Comparison

H_0: $\mu_{1Coop} = \mu_{2coop}$
H_1: $\mu_{1Coop} \neq \mu_{2coop}$

Male vs. Female in Cooperative Learning condition:

$$\text{t test} = \frac{(85-75)}{se_{difference}}$$

Lecture Comparison

H_0: $\mu_{1Lecture} = \mu_{2Lecture}$
H_1: $\mu_{1Lecture} \neq \mu_{2Lecture}$

Male vs. Female in Lecture condition:

$$\text{t test} = \frac{(80-80)}{se_{difference}}$$

Self-paced Comparison

H_0: $\mu_{1Self} = \mu_{2Self}$
H_1: $\mu_{1Self} \neq \mu_{2Self}$

Male vs. Female in Self-paced condition:

$$\text{t test} = \frac{(75-85)}{se_{difference}}$$

(c.2) Instruction Simple Main Effects

Requires multiple comparisons separately for each sex:

Comparisons for Females

Comparison	Adjusted Mean Difference	Standard Error of Mean Difference	95% CI
Coop vs. Self	10		
Coop vs. Lecture	5		
Self vs. Lecture	-5		

Comparisons for Males

Comparison	Adjusted Mean Difference	Standard Error of Mean Difference	95% CI
Coop vs. Self	-10		
Coop vs. Lecture	-5		
Self vs. Lecture	5		

5. ANOVA Computation

As before, ANOVA computation is based upon the information found in the summary table below.

Table 4
Two-way ANOVA Summary Table

Source	SS	df	MS	F
Factor A	SS_A	$df_A = j - 1$	SS_A/df_A	MS_A/MS_w
Factor B	SS_B	$df_B = k - 1$	SS_B/df_B	MS_B/MS_w
Interaction A×B	SS_{AB}	$df_{AB} = (j - 1)(k-1)$	SS_{AB}/df_{AB}	MS_{AB}/MS_w
Within Error	SS_W	$df_w = jk(n-1)$	SS_w/df_w	
total	SS_T	$df_t = n - 1$		

6. SPSS Results for Sample Data

SPSS results of the two-way ANOVA are provided below. The GENERAL LINEAR MODEL->UNIVARIATE command was used, and a model with interaction was specified (this is done automatically in SPSS). This will be illustrated during the chat.

To obtain simple main effect comparisons will require use of SPSS syntax commands since the Window's pull-down menus don't contain this option (at least with my version of SPSS). SPSS Syntax (commands) and the part added in **bold and underlined**:

```
UNIANOVA
  achievement  BY instruction sex
  /METHOD = SSTYPE(3)
  /INTERCEPT = INCLUDE
  /EMMEANS = TABLES(instruction) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(sex) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(instruction*sex) Compare (instruction) ADJ(BONFERRONI)
  /EMMEANS = TABLES(instruction*sex) Compare (sex)
  /PRINT = DESCRIPTIVE
  /CRITERIA = ALPHA(.05)
  /DESIGN = instruction sex instruction*sex .
```

To keep output simple, better not to seek output showing comparisons for main effects, so the lines in gray and with strikeout can be deleted:

```
UNIANOVA
  achievement  BY instruction sex
  /METHOD = SSTYPE(3)
  /INTERCEPT = INCLUDE
  /EMMEANS = TABLES(instruction) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(sex) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(instruction*sex) Compare (instruction) ADJ(BONFERRONI)
  /EMMEANS = TABLES(instruction*sex) Compare (sex)
  /PRINT = DESCRIPTIVE
  /CRITERIA = ALPHA(.05)
  /DESIGN = instruction sex instruction*sex .
```

Results are presented below.

Descriptive Statistics

instruction	sex	Mean	Std. Deviation	N
Co-operative	f	85.0000	1.41421	2
	m	75.0000	1.41421	2
	Total	80.0000	5.88784	4
Lecture	f	80.0000	1.41421	2
	m	80.0000	2.82843	2
	Total	80.0000	1.82574	4
Self-paced	f	75.0000	1.41421	2
	m	85.0000	1.41421	2
	Total	80.0000	5.88784	4
Total	f	80.0000	4.60435	6
	m	80.0000	4.73286	6
	Total	80.0000	4.45176	12

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	200.000(a)	5	40.000	13.333	.003
Intercept	76800.000	1	76800.000	25600.000	.000
instruction	.000	2	.000	.000	1.000
sex	.000	1	.000	.000	1.000
instruction * sex	200.000	2	100.000	33.333	.001
Error	18.000	6	3.000		
Total	77018.000	12			
Corrected Total	218.000	11			

a R Squared = .917 (Adjusted R Squared = .849)

Pairwise Comparisons

Dependent Variable: achievement

sex	(I) instruction	(J) instruction	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
						Lower Bound	Upper Bound
f	Co-operative	Lecture	5.000	1.732	.083	-.694	10.694
		Self-paced	10.000(*)	1.732	.004	4.306	15.694
	Lecture	Co-operative	-5.000	1.732	.083	-10.694	.694
		Self-paced	5.000	1.732	.083	-.694	10.694
	Self-paced	Co-operative	-10.000(*)	1.732	.004	-15.694	-4.306
		Lecture	-5.000	1.732	.083	-10.694	.694
m	Co-operative	Lecture	-5.000	1.732	.083	-10.694	.694
		Self-paced	-10.000(*)	1.732	.004	-15.694	-4.306
	Lecture	Co-operative	5.000	1.732	.083	-.694	10.694
		Self-paced	-5.000	1.732	.083	-10.694	.694
	Self-paced	Co-operative	10.000(*)	1.732	.004	4.306	15.694
		Lecture	5.000	1.732	.083	-.694	10.694

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.

Pairwise Comparisons

Dependent Variable: achievement

instruction	(I) sex	(J) sex	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
						Lower Bound	Upper Bound
Co-operative	f	m	10.000(*)	1.732	.001	5.762	14.238
	m	f	-10.000(*)	1.732	.001	-14.238	-5.762
Lecture	f	m	.000	1.732	1.000	-4.238	4.238
	m	f	.000	1.732	1.000	-4.238	4.238
Self-paced	f	m	-10.000(*)	1.732	.001	-14.238	-5.762
	m	f	10.000(*)	1.732	.001	5.762	14.238

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

The above two tables show simple main effects – comparisons of categories in one IV per levels of categories of a second IV. For example, for each level of sex, the possible pairwise comparisons of instruction are present per sex:

For males the follow pairwise comparisons are provided:

Co-operative	Lecture
	Self-paced
Lecture	Co-operative
	Self-paced
Self-paced	Co-operative
	Lecture

And for females, the same pairwise comparisons are provided. If main effects for instruction were examined, the pairwise comparisons for instruction would be taken across both sexes simultaneously as marginal means, not separately as means per sex.

5. APA Styled Presentation

Table 5
ANOVA Results and Descriptive Statistics for Achievement by Student Sex and Instruction Type

Variable	Mean	SD	n
Female			
Coop	85.00	1.41	2
Self	75.00	1.41	2
Lecture	80.00	1.41	2
Male			
Coop	75.00	1.41	2
Self	85.00	1.41	2
Lecture	80.00	1.41	2

Source	SS	df	MS	F
Instruction (I)	0.00	2	0.00	0.00
Sex (S)	0.00	1	0.00	0.00
I×S	200.00	2	100.00	33.33*
Error	18.00	6	3.00	

Note: R^2 = .92, adj. R^2 = .85. Coop = co-operative learning, Self = self-paced, and Lecture = lecture instruction.
* $p < .05$

Table 6
Comparisons of Mean Differences in Achievement by Instruction and Student Sex

Comparison by Student Sex	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
Females			
Coop vs. Lecture	5.00	1.73	-0.69, 10.69
Coop vs. Self	10.00*	1.73	4.31, 15.69
Lecture vs. Self	5.00	1.73	-0.69, 10.69
Males			
Coop vs. Lecture	-5.00	1.73	-10.69, 0.69
Coop vs. Self	-10.00*	1.73	-15.69, -4.31
Lecture vs. Self	-5.00	1.73	-10.69, 0.69

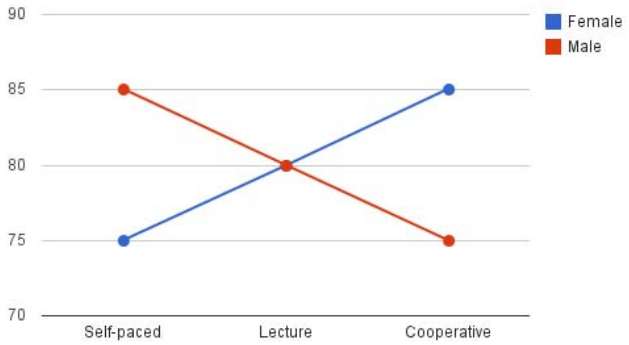
Note: Coop = co-operative learning, Self = self-paced, And Lecture = lecture instruction.
* $p < .05$, where p-values are adjusted using the Bonferroni method.

Table 7
Comparisons of Mean Differences in Achievement by Student Sex and Instruction

Comparison by Instruction	Estimated Mean Difference	Standard Error of Difference	95% CI
Cooperative Learning			
Female vs. Male	10.00*	1.73	5.76, 14.24
Lecture			
Female vs. Male	0.00	1.73	-4.24, 4.24
Self-paced			
Female vs. Male	-10.00*	1.73	-14.24, -5.76

Note: Coop = co-operative learning, Self = self-paced, And Lecture = lecture instruction.
* $p < .05$.

Figure 4
Achievement Interaction between Student Sex and Instruction Type



ANOVA results show that achievement is associated with student sex and instruction type with a statistically significant interaction between the two factors. The interaction can be seen in Figure 4. Table 6 shows that for females the only statistical difference in performance occurs between cooperative learning and self-paced instruction, and the same is true for males. However, the differences are reversed for the two sexes. As indicated in Table 7 males perform better in self-paced instruction, females perform better in cooperative learning, and they perform similarly in the lecture condition.

6. Regression Results

To model these data in regression it is first required that dummy variables be formed. Two for instruction, Coop (1 = coop, 0 = others) and Self (1 = self-paced, 0 = others), are formed which leaves lecture as the reference group. One is formed for student sex with Female = 1 and 0 = male. Next one must create the interaction terms. There are two predictors, instruction and sex, and since there are two dummy variables for instruction, two interactions are needed, one for the Coop dummy and one for the Self dummy. They are created as follows:

Interaction: Coop*Female = Coop × Female, and
Interaction: Lecture*Female = Lecture × Female.

The corresponding regression model is

$Y' = b_0 + b_1 coop + b_2 self + b_3 female + b_4 coop \times female + b_5 self \times female$

Data for this regression model appears in Table 8 below.

Table 8: Sample Data Coded for Regression Analysis

Achievement	Instruction	Student Sex	Coop	Lecture	Female	Coop*Female	Lecture*Female
74.00	Co-operative	m	1.00	.00	.00	.00	.00
76.00	Co-operative	m	1.00	.00	.00	.00	.00
84.00	Co-operative	f	1.00	.00	1.00	1.00	.00
86.00	Co-operative	f	1.00	.00	1.00	1.00	.00
78.00	Lecture	m	.00	.00	.00	.00	.00
82.00	Lecture	m	.00	.00	.00	.00	.00
79.00	Lecture	f	.00	.00	1.00	.00	.00
81.00	Lecture	f	.00	.00	1.00	.00	.00
86.00	Self-paced	m	.00	1.00	.00	.00	.00
84.00	Self-paced	m	.00	1.00	.00	.00	.00
76.00	Self-paced	f	.00	1.00	1.00	.00	1.00
74.00	Self-paced	f	.00	1.00	1.00	.00	1.00

SPSS regression command is specified below.

```
REGRESSION
  /DESCRIPTIVES MEAN STDDEV CORR SIG N
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS CI R ANOVA CHANGE
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT achievement
  /METHOD=TEST (coop self) (female) (interact_coopXfemale interact_selfXfemale ) .
```

Note the Method=TEST command above has each group of variables and dummy variables collected in parentheses. This tells SPSS to computer ΔR^2 values and partial F-tests for separately for instruction (coop self), sex (female), and the interaction between instruction and sex (interact_coopXfemale interact_selfXfemale). This METHOD=TEST command can only be accessed through SPSS Syntax using the Paste button in the Regression window. Results are posted below.

Descriptive Statistics

	Mean	Std. Deviation	N
achievement	80.0000	4.45176	12
coop	.3333	.49237	12
self	.3333	.49237	12
female	.5000	.52223	12
interact_coopXfemale	.1667	.38925	12
interact_selfXfemale	.1667	.38925	12

Correlations

		achievement	coop	self	female	interact_coopXfemale	interact_selfXfemale
Pearson Correlation	achievement	1.000	.000	.000	.000	.525	-.525
	coop	.000	1.000	-.500	.000	.632	-.316
	self	.000	-.500	1.000	.000	-.316	.632
	female	.000	.000	.000	1.000	.447	.447
	interact_coopXfemale	.525	.632	-.316	.447	1.000	-.200
	interact_selfXfemale	-.525	-.316	.632	.447	-.200	1.000
Sig. (1-tailed)	achievement	.	.500	.500	.500	.040	.040
	coop	.500	.	.049	.500	.014	.158
	self	.500	.049	.	.500	.158	.014
	female	.500	.500	.500	.	.072	.072
	interact_coopXfemale	.040	.014	.158	.072	.	.267
	interact_selfXfemale	.040	.158	.014	.072	.267	.
N	achievement	12	12	12	12	12	12
	coop	12	12	12	12	12	12
	self	12	12	12	12	12	12
	female	12	12	12	12	12	12
	interact_coopXfemale	12	12	12	12	12	12
	interact_selfXfemale	12	12	12	12	12	12

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.958(a)	.917	.849	1.73205	.917	13.333	5	6	.003

a Predictors: (Constant), interact_selfXfemale, interact_coopXfemale, self, coop, female

Note the ANOVA table below presents the ΔR^2 values and partial F-tests. The ΔR^2 values can be found in the last column labeled "R Square Change" and their corresponding partial F-tests are in the "F" column. A curious result provides some discrepancy between regression results and ANOVA results. Note the partial F-test for instruction (i.e., Coop and Self) is F = 16.667 and ΔR^2 = .459. In ANOVA both F and ΔR^2 are 0.00. This discrepancy is the result of how ΔR^2 is calculated in regression—order of entry of variables and the increment to R^2 value—which is not a factor in ANOVA given the SS calculations. In regression one normally enters the relevant variables before entering the product terms (interactions), and reversing the order can produce misleading results. That is what happened here. The overal model and end result, however is the same. For example, the model R^2 is .917 in both ANOVA and regression, and the model F ratio is 13.33 in both regression and ANOVA. Similarly, the SEE in both is 1.73205 and the MSE is 3.00 in both. Lastly, the parameter estimates for the equation, given below in the "Coefficients' table for regression matches the coefficient that can be specified in output for ANOVA.

ANOVA(c)

Model			Sum of Squares	df	Mean Square	F	Sig.	R Square Change
1	Subset Tests	coop, self	100.000	2	50.000	16.667	.004(a)	.459
		female	.000	1	.000	.000	1.000(a)	.000
		interact_coopXfemale, interact_selfXfemale	200.000	2	100.000	33.333	.001(a)	.917
	Regression		200.000	5	40.000	13.333	.003(b)	
	Residual		18.000	6	3.000			
	Total		218.000	11				

a Tested against the full model.
b Predictors in the Full Model: (Constant), interact_selfXfemale, interact_coopXfemale, self, coop, female.
c Dependent Variable: achievement

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	80.000	1.225		65.320	.000	77.003	82.997
	coop	-5.000	1.732	-.553	-2.887	.028	-9.238	-.762
	self	5.000	1.732	.553	2.887	.028	.762	9.238
	female	-9.268E-16	1.732	.000	.000	1.000	-4.238	4.238
	interact_coopXfemale	10.000	2.449	.874	4.082	.006	4.006	15.994
	interact_selfXfemale	-10.000	2.449	-.874	-4.082	.006	-15.994	-4.006

a Dependent Variable: achievement

7. Exercises

(a) Does student mathematics motivation vary by teacher and workbook? Students in each of three teacher's were randomly assigned to either mathematics workbook A, B, or C, so in each class each of the three workbooks was used by different students. Motivation scores derived from a scale that varies from 5 (low) to 15 (high).

Table 7a
Mathematics Motivation by Workbook and Teacher

Motivation	Workbook	Teacher
15.00	A	Smith
13.00	A	Smith
12.00	B	Smith
10.00	B	Smith
9.00	C	Smith
8.00	C	Smith
13.00	A	Griffin
12.00	A	Griffin
6.00	B	Griffin
8.00	B	Griffin
8.00	C	Griffin
7.00	C	Griffin
9.00	A	Marshall
7.00	A	Marshall
8.00	B	Marshall
7.00	B	Marshall
8.00	C	Marshall
6.00	C	Marshall

(b) My son recently completed a science fair project to examine the possible impact of household cleaners on plant growth. The experiment incorporated two factors: Plant Species (Corn, Green Beans, and Radishes) and Watering Condition (Nature's Source Green Cleaner, Lysol, and Water-only). Briefly described, there were three watering conditions. In one plants were exposed to water in which Nature's Source Green Cleaner was added, in a second Lysol was added to water, and in the control condition no cleaner was added to water. For both experimental conditions, one tablespoon of cleaner was added per gallon of water. This watering treatment tested on three types of garden plants: Corn, Green Beans, and Radishes. One outcome of interest was the height, in centimeters (CM), of each plant 6 weeks after the experiment commenced. Those data are recorded below. The research question of interest is whether watering condition affected plant growth. Not of interest is whether plant height differs by species because this is known and assumed, so if an interaction occurs, only the comparison of watering conditions will be needed.

Table 7b
Height of Plants in Centimeters

	Watering Condition		
Plant Species	Green Cleaner	Lysol	Water Only
Corn	29, 26, 25, 26, 14, 20, 21, 21.5, 21	26, 26, 29, 32, 27.5, 33, 19, 22, 30	25.7, 27, 28.5, 24, 24.5, 24, 25, 27, 25.5
Green Beans	23, 25, 21, 24, 25, 22, 22.5, 22.7, 18	20, 16, 14, 18, 22, 20, 25, 22, 20	28.5, 22, 25, 21.5, 18, 25, 25, 24.5, 23.7
Radishes	10.5, 12.5, 9.7, 14, 13, 12.5, 13, 10.5, 14.5	7, 8, 14, 14, 13.5, 10, 15, 15.5, 12	14.5, 11, 10, 14, 12, 14, 14, 12.5, 16

For these data to be analyzed in most statistical software the raw scores listed above must be formatted in a column and row layout similar to that displayed below

Height	Plant Species	Treatment Condition
29	Corn	Green
26	Corn	Green
25	Corn	Green
26	Corn	Green
14	Corn	Green
20	Corn	Green
21	Corn	Green
21.5	Corn	Green
21	Corn	Green
26	Corn	Lysol
26	Corn	Lysol
29	Corn	Lysol
32	Corn	Lysol
Etc.	Etc.	Etc.

These data can be accessed in the above format here:

http://tinyurl.com/3afh2vv

or

https://spreadsheets.google.com/ccc?key=0AoKw33oyzB1NdFRFSmlOLXVsQVRKdHZ5Umd4bGl3WUE&hl=en&authkey=CLGP06oF

8. Exercise Answers

7a Motivation by Workbook and Instructor

SPSS syntax commands to obtain simple main effect comparisons. The sections underlined and in bold were added manually to obtain the simple main effects.

```
UNIANOVA
  motivaton  BY workbook teacher
  /METHOD = SSTYPE(3)
  /INTERCEPT = INCLUDE
  /EMMEANS = TABLES(workbook*teacher) COMPARE(workbook) ADJ(BONFERRONI)
  /EMMEANS = TABLES(workbook*teacher) COMPARE(teacher) ADJ(BONFERRONI)
  /PRINT = DESCRIPTIVE
  /CRITERIA = ALPHA(.05)
  /DESIGN = workbook teacher workbook*teacher.
```

Descriptive Statistics

Dependent Variable: motivaton

workbook	teacher	Mean	Std. Deviation	N
A	Griffin	12.5000	.70711	2
	Marshall	8.0000	1.41421	2
	Smith	14.0000	1.41421	2
	Total	11.5000	2.94958	6
B	Griffin	7.0000	1.41421	2
	Marshall	7.5000	.70711	2
	Smith	11.0000	1.41421	2
	Total	8.5000	2.16795	6
C	Griffin	7.5000	.70711	2
	Marshall	7.0000	1.41421	2
	Smith	8.5000	.70711	2
	Total	7.6667	1.03280	6
Total	Griffin	9.0000	2.82843	6
	Marshall	7.5000	1.04881	6
	Smith	11.1667	2.63944	6
	Total	9.2222	2.66912	18

Tests of Between-Subjects Effects

Dependent Variable: motivaton

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	109.111(a)	8	13.639	10.229	.001
Intercept	1530.889	1	1530.889	1148.167	.000
workbook	48.778	2	24.389	18.292	.001
teacher	40.778	2	20.389	15.292	.001
workbook * teacher	19.556	4	4.889	3.667	.049
Error	12.000	9	1.333		
Total	1652.000	18			
Corrected Total	121.111	17			

a R Squared = .901 (Adjusted R Squared = .813)

The table below provides simple main effects for workbook for each teacher.

Pairwise Comparisons

Dependent Variable: motivaton

teacher	(I) workbook	(J) workbook	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
						Lower Bound	Upper Bound
Griffin	A	B	5.500(*)	1.155	.003	2.113	8.887
		C	5.000(*)	1.155	.006	1.613	8.387
	B	A	-5.500(*)	1.155	.003	-8.887	-2.113
		C	-.500	1.155	1.000	-3.887	2.887
	C	A	-5.000(*)	1.155	.006	-8.387	-1.613
		B	.500	1.155	1.000	-2.887	3.887
Marshall	A	B	.500	1.155	1.000	-2.887	3.887
		C	1.000	1.155	1.000	-2.387	4.387
	B	A	-.500	1.155	1.000	-3.887	2.887
		C	.500	1.155	1.000	-2.887	3.887
	C	A	-1.000	1.155	1.000	-4.387	2.387
		B	-.500	1.155	1.000	-3.887	2.887
Smith	A	B	3.000	1.155	.086	-.387	6.387
		C	5.500(*)	1.155	.003	2.113	8.887
	B	A	-3.000	1.155	.086	-6.387	.387
		C	2.500	1.155	.176	-.887	5.887
	C	A	-5.500(*)	1.155	.003	-8.887	-2.113
		B	-2.500	1.155	.176	-5.887	.887

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.

The table below provides simple main effects for teacher for each level of workbook.

Pairwise Comparisons

Dependent Variable: motivaton

workbook	(I) teacher	(J) teacher	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
						Lower Bound	Upper Bound
A	Griffin	Marshall	4.500(*)	1.155	.011	1.113	7.887
		Smith	-1.500	1.155	.679	-4.887	1.887
	Marshall	Griffin	-4.500(*)	1.155	.011	-7.887	-1.113
		Smith	-6.000(*)	1.155	.002	-9.387	-2.613
	Smith	Griffin	1.500	1.155	.679	-1.887	4.887
		Marshall	6.000(*)	1.155	.002	2.613	9.387
B	Griffin	Marshall	-.500	1.155	1.000	-3.887	2.887
		Smith	-4.000(*)	1.155	.021	-7.387	-.613
	Marshall	Griffin	.500	1.155	1.000	-2.887	3.887
		Smith	-3.500(*)	1.155	.043	-6.887	-.113
	Smith	Griffin	4.000(*)	1.155	.021	.613	7.387
		Marshall	3.500(*)	1.155	.043	.113	6.887
C	Griffin	Marshall	.500	1.155	1.000	-2.887	3.887
		Smith	-1.000	1.155	1.000	-4.387	2.387
	Marshall	Griffin	-.500	1.155	1.000	-3.887	2.887
		Smith	-1.500	1.155	.679	-4.887	1.887
	Smith	Griffin	1.000	1.155	1.000	-2.387	4.387
		Marshall	1.500	1.155	.679	-1.887	4.887

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.

APA Results for 7a

Table 7a.1
Descriptive Statistics of Motivation by Instructor and Workbook

	Instructor								
	Smith			Griffin			Marshall		
Workbook	M	SD	n	M	SD	n	M	SD	n
A	14.00	1.41	2	12.50	0.71	2	8.00	1.41	2
B	11.00	1.41	2	7.00	1.41	2	7.50	0.71	2
C	8.50	0.71	2	7.50	0.71	2	7.00	1.41	2

Note: N = 18, Grand M = 9.22, SD = 2.67

Table 7a.2
ANOVA Summary for Motivation by Instructor and Workbook

Source	SS	df	MS	F
Workbook	48.78	2	24.39	18.29*
Instructor	40.78	2	20.39	15.29*
I × W	19.56	4	4.89	3.67*
Error	12.00	9	1.33	

Note: $R^2 = .90$, adj. $R^2 = .81$.
* $p < .05$

Table 7a.3
Comparisons of Mean Differences in Motivation by Workbook and Instructor

Comparison by Instructor	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
Griffin			
A vs. B	5.50*	1.16	2.11, 8.89
A vs. C	5.00*	1.16	1.61, 8.39
B vs. C	-0.50	1.16	-3.89, 2.89
Marshall			
A vs. B	0.50	1.16	-2.89, 3.89
A vs. C	1.00	1.16	-2.39, 4.39
B vs. C	0.50	1.16	-2.89, 3.89
Smith			
A vs. B	3.00	1.16	-0.39, 6.39
A vs. C	5.50*	1.16	2.11, 8.89
B vs. C	2.50	1.16	-0.89, 5.89

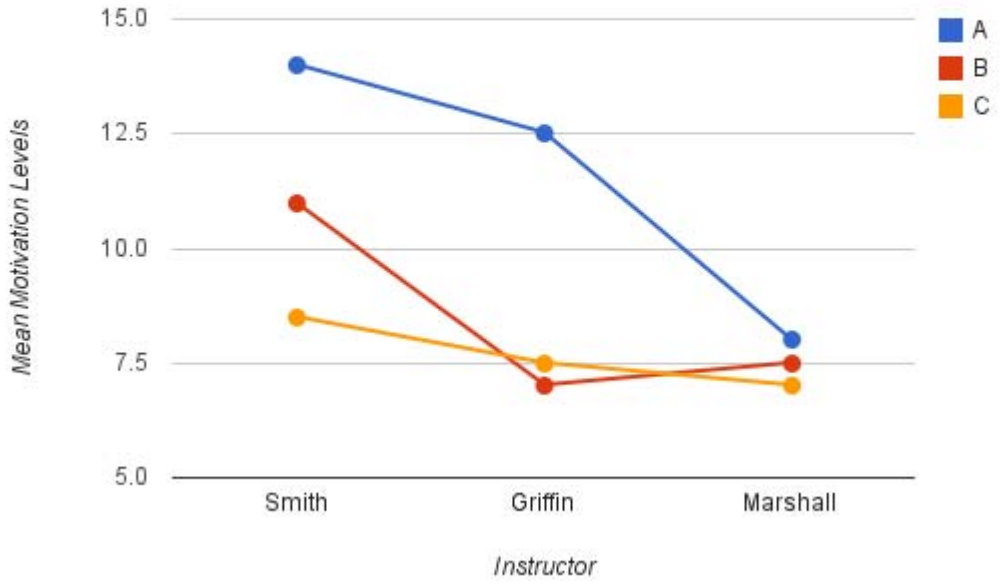
* $p < .05$, where p-values are adjusted using the Bonferroni method.

Table 7a.4
Comparisons of Mean Differences in Motivation by Instructor and Workbook

	Comparison by Workbook	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
A				
	Griffin vs. Marshall	4.50*	1.16	1.11, 7.89
	Griffin vs. Smith	-1.50	1.16	-4.89, 1.89
	Marshall vs. Smith	-6.00*	1.16	-9.39, -2.61
B				
	Griffin vs. Marshall	-0.50	1.16	-3.89, 2.89
	Griffin vs. Smith	-4.00*	1.16	-7.39, -0.61
	Marshall vs. Smith	-3.50*	1.16	-6.89, -0.11
C				
	Griffin vs. Marshall	0.50	1.16	-2.89, 3.89
	Griffin vs. Smith	-1.00	1.16	-4.39, 2.39
	Marshall vs. Smith	-1.50	1.16	-4.89, 1.89

* $p < .05$, where p-values are adjusted using the Bonferroni method.

Figure 4
Motivation by Instructor and Workbook



Results of the ANOVA show a statistically significant interaction between instructor and workbook and this indicates that mean differences in motivation exist by both instructor and workbook. Simple main effect comparisons are provided in Tables 7a.3 and 7a.4., and a plot of means appears in Figure 4. When examining Marshall's class, there were no differences in mean motivation levels for students using workbooks A, B, or C as illustrated in Figure 4. In Griffin's class, students using workbooks B and C displayed similar levels of motivation, however, students with workbook A recorded statistically higher levels of motivation than students with either workbooks B or C. Lastly, students in Smith's class demonstrated highest levels with workbook A and lowest with workbook C; those using workbook B displayed levels of motivation placed between A and C.

7b Plant Height by Watering Condition and Species

SPSS syntax commands to obtain simple main effect comparisons for watering condition. The sections underlined and in bold were added manually to obtain the simple main effects.

```
UNIANOVA
  height  BY species condition
  /METHOD = SSTYPE(3)
  /INTERCEPT = INCLUDE
  /PLOT = PROFILE( species*condition condition*species )
  /EMMEANS = TABLES(OVERALL)
  /EMMEANS = TABLES(species) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(condition) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(species*condition) COMPARE(condition) ADJ(BONFERRONI)
  /PRINT = DESCRIPTIVE
  /CRITERIA = ALPHA(.05)
  /DESIGN = species condition species*condition .
```

Descriptive Statistics

Dependent Variable: height

species	condition	Mean	Std. Deviation	N
Beans	Control	23.6889	2.93617	9
	Green	22.5778	2.17185	9
	Lysol	19.6667	3.31662	9
	Total	21.9778	3.23661	27
Corn	Control	25.6889	1.53984	9
	Green	22.6111	4.42844	9
	Lysol	27.1667	4.54148	9
	Total	25.1556	4.10481	27
Radishes	Control	13.1111	1.88378	9
	Green	12.2444	1.65840	9
	Lysol	12.1111	3.09008	9
	Total	12.4889	2.25394	27
Total	Control	20.8296	6.00832	27
	Green	19.1444	5.74914	27
	Lysol	19.6481	7.20419	27
	Total	19.8741	6.31246	81

Tests of Between-Subjects Effects

Dependent Variable: height

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2525.433(a)	8	315.679	34.316	.000
Intercept	31993.284	1	31993.284	3477.834	.000
species	2345.236	2	1172.618	127.470	.000
condition	40.405	2	20.203	2.196	.119
species * condition	139.793	4	34.948	3.799	.007
Error	662.342	72	9.199		
Total	35181.060	81			
Corrected Total	3187.776	80			

a R Squared = .792 (Adjusted R Squared = .769)

Estimates

Dependent Variable: height

species	condition	Mean	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
Beans	Control	23.689	1.011	21.673	25.704
	Green	22.578	1.011	20.562	24.593
	Lysol	19.667	1.011	17.651	21.682
Corn	Control	25.689	1.011	23.673	27.704
	Green	22.611	1.011	20.596	24.627
	Lysol	27.167	1.011	25.151	29.182
Radishes	Control	13.111	1.011	11.096	15.127
	Green	12.244	1.011	10.229	14.260
	Lysol	12.111	1.011	10.096	14.127

Pairwise Comparisons

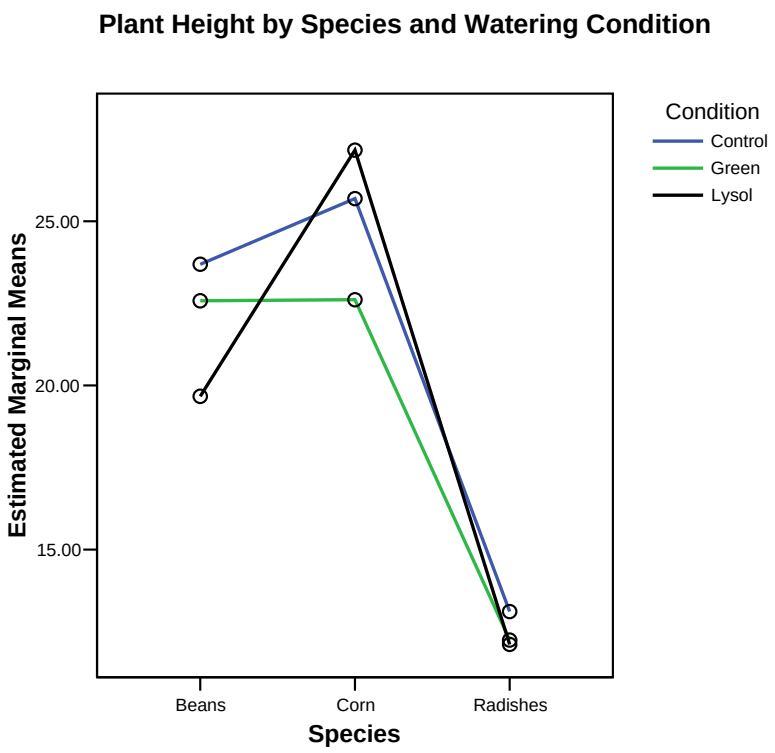
Dependent Variable: height

species	(I) condition	(J) condition	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
						Lower Bound	Upper Bound
Beans	Control	Green	1.111	1.430	1.000	-2.394	4.616
		Lysol	4.022(*)	1.430	.019	.518	7.527
	Green	Control	-1.111	1.430	1.000	-4.616	2.394
		Lysol	2.911	1.430	.136	-.594	6.416
	Lysol	Control	-4.022(*)	1.430	.019	-7.527	-.518
		Green	-2.911	1.430	.136	-6.416	.594
Corn	Control	Green	3.078	1.430	.104	-.427	6.582
		Lysol	-1.478	1.430	.914	-4.982	2.027
	Green	Control	-3.078	1.430	.104	-6.582	.427
		Lysol	-4.556(*)	1.430	.006	-8.060	-1.051
	Lysol	Control	1.478	1.430	.914	-2.027	4.982
		Green	4.556(*)	1.430	.006	1.051	8.060
Radishes	Control	Green	.867	1.430	1.000	-2.638	4.371
		Lysol	1.000	1.430	1.000	-2.505	4.505
	Green	Control	-.867	1.430	1.000	-4.371	2.638
		Lysol	.133	1.430	1.000	-3.371	3.638
	Lysol	Control	-1.000	1.430	1.000	-4.505	2.505
		Green	-.133	1.430	1.000	-3.638	3.371

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.



APA Results for 7b

Table 7b.1
Descriptive Statistics of Plant Height by Watering Condition and Species

	Watering Condition								
	Green Cleaner			Lysol			Water Only		
Plant Species	M	SD	n	M	SD	n	M	SD	n
Beans	22.58	2.17	9	19.67	3.32	9	23.69	2.94	9
Corn	22.61	4.43	9	27.17	4.54	9	25.69	1.54	9
Radishes	12.24	1.66	9	12.11	3.09	9	13.11	1.88	9

Note: N = 81, Grand M = 19.87, SD = 6.31

Table 7b.2
ANOVA Summary for Plant Height by Watering Condition and Plant Species

Source	SS	df	MS	F
Species	2345.24	2	1172.62	127.47*
Condition	40.41	2	20.20	2.20
S × C	139.79	4	34.95	3.80*
Error	662.34	72	9.20	

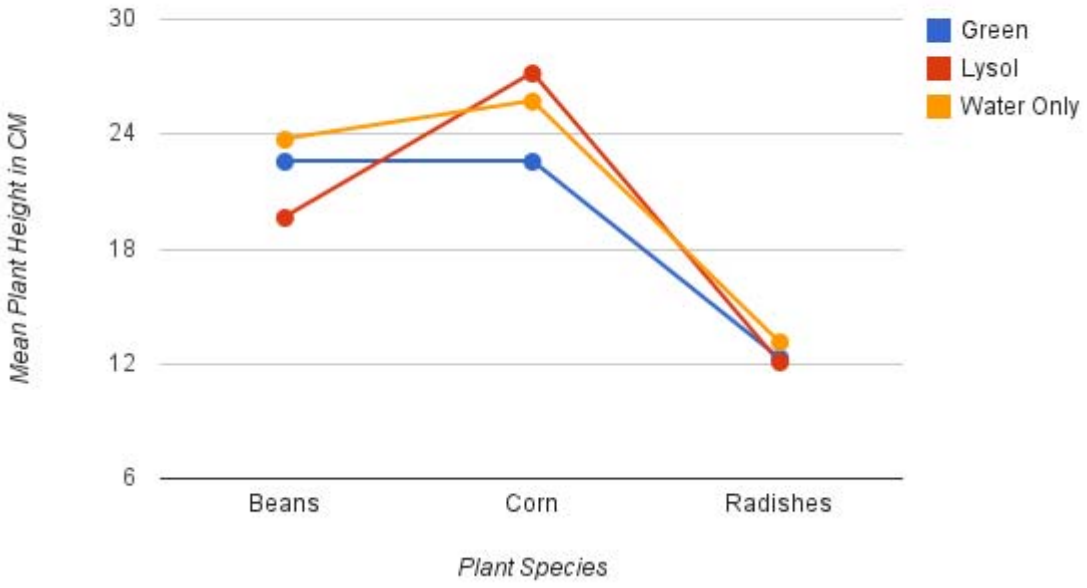
Note: R^2 = .79, adj. R^2 = .77.
* $p < .05$

Table 7b.3
Comparisons of Mean Differences in Plant Height Watering Condition and Species

Comparison by Instructor	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
Beans			
Control vs. Green	1.11	1.43	-2.39, 4.62
Control vs. Lysol	4.02*	1.43	0.52, 7.53
Green vs. Lysol	2.91	1.43	-0.59, 6.42
Corn			
Control vs. Green	3.08	1.43	-0.43, 6.58
Control vs. Lysol	-1.48	1.43	-4.98, 2.03
Green vs. Lysol	-4.56*	1.43	-8.06, -1.05
Radishes			
Control vs. Green	0.87	1.43	-2.64, 4.37
Control vs. Lysol	1.00	1.43	-2.51, 4.51
Green vs. Lysol	0.13	1.43	-3.37, 3.64

* $p < .05$, where p-values are adjusted using the Bonferroni method.

Figure 4
Plant Height by Watering Condition and Plant Species



A two-way interaction exists between watering condition and plant species. Figure 4 shows plant height means for cell combination of species and condition. For radishes there we no statistical differences in plant heights. For corn the tallest plants were those in the Lysol condition, followed by water only and then Green Cleaner. The mean difference between those in the Lysol and Green Cleaner conditions was significant. This was unexpected since the water only condition was predicted to produce the tallest plants. For Beans the pattern of means was consistent with expectations: Water only plants were tallest, followed next by Green Cleaner condition, with Lysol condition plants the shortest. The difference between water only and Lysol conditions was statistically significant.