Notes 9b: Two-way and Multi-way ANOVA (without Interactions)

1. Two-way ANOVA

Two-way ANOVA is simply ANOVA with two qualitative independent variables. For example, if one wanted to know if type of instruction (e.g., cooperative learning, self-paced, or lecture) and sex is associated with student achievement, two-way ANOVA would be appropriate. The multiple regression model for a two-way ANOVA looks like this:

$Y' = b_0 + b_1 coop + b_2 self + b_3 female$

where "coop," "self," and "female" are dummy variables and the omitted categories (reference group) are males in the lecture treatment. Example data are provided below.

Table 1
Example Date for Two-way ANOVA

Achievement	Instruction Type	Sex
78	coop	m
79	coop	m
71	coop	f
73	coop	f
88	self	m
86	self	m
83	self	f
80	self	f
91	lecture	m
89	lecture	m
88	lecture	f
87	lecture	f

2. Hypotheses

The hypotheses remain essentially unchanged from previously with ANOVA except comparisons denote taking means over categories, or cells, of the second variable:

(a) Main Effect Test – compare means across J levels of first variable (J row population means)

Null: H_0: general form $\mu_{1.} = \mu_{2.} = \ldots = \mu_{J.}$
Non-directional alternative H_a: not all means of J levels are equal

Where the subscript period, ., represents taking the mean across all levels of the second variable.

Current example for instruction:

H_0: $\mu_{1.} = \mu_{2.} = \mu_{3.}$ (taking mean across both males and females for each category of instruction)
H_a: not all of the instructional treatments means are equal

(b) Main Effect Test – compare means across K levels of second independent variable (K column population means)

Null: H_0: general form $\mu_{.1} = \mu_{.2} = \ldots = \mu_{.k.}$
Non-directional alternative H_a: not all means of K levels are equal

Current example for sex:

H_0: $\mu_{1.} = \mu_{2.}$ (taking mean across all levels of instruction)
H_a: $\mu_{1.} \neq \mu_{2.}$.

(c) Test interaction between the two independent variables

Null: H_0: general form all $(\mu_{jk} - \mu_{j.} - \mu_{.k} + \mu) = 0$
Non-directional alternative H_a: all interaction $\alpha\beta = 0$

If the interaction is statistically significant, then focus will be upon not Main Effects tests, but upon Simple Main Effects tests. This will be covered in "Notes 9c Two-way ANOVA with Interactions"

Table 2
Illustration of Hypotheses for Sample Data (Achievement Means Recorded)

	Coop	Lecture	Self	Marginal Means for Sex
Female	72.00	87.50	81.50	80.33
Male	78.50	90.00	87.00	85.17
Marginal Means for Instruction	75.25	88.75	84.25	

The null for instruction is H_0: $\mu_{1.} = \mu_{2.} = \mu_{3.}$ and refers to the marginal means of 75.25, 88.75, and 84.25 above.

The null for sex H_0: $\mu_{1.} = \mu_{2.}$ references the marginal sex means of 80.33 and 85.17.

Mean differences could also be hypothesized. For example for sex, the null mean difference would be:

H_0: $\mu_{1.} - \mu_{2.} = 0.00$

To find the marginal mean difference, one could simply use the marginal means and find the mean difference as follows:

$M_{female} - M_{male} = 80.33 - 85.17 = -4.83$

Another approach is to calculate the mean difference for each category of instruction and take the mean of these mean differences:

Table 3
Marginal Mean Sex Difference

	Coop	Lecture	Self	
Female	72.00	87.50	81.50	
Male	78.50	90.00	87.00	Mean of Mean Differences:
Mean difference:	-6.50	-2.50	-5.50	-4.83

The point of the above illustration is to show that *main effect* hypotheses tests examine marginal means, and marginal mean differences may not be the same across every category of the second IV. Note that the marginal mean difference for sex varies from -2.50 for lecture to -6.50 for coop.

Graph of data can be seen here, and this helps show that marginal mean differences vary by instruction:

http://tinyurl.com/38luthp

or here

https://spreadsheets.google.com/ccc?key=0AoKw33oyzB1NdG03REZmNmdPTzFzc3I4cE1ITmM5Smc&hl=en&authkey=CMzqsqwD

3. ANOVA Computation

As before, ANOVA computation is based upon the information found in the summary table below.

Table 4
Two-way ANOVA Summary Table

Source	SS	df	MS	F
Factor A	SS_A	$df_A = j - 1$	SS_A/df_A	MS_A/MS_w
Factor B	SS_B	$df_B = k - 1$	SS_B/df_B	MS_B/MS_w
Interaction A×B	SS_{AB}	$df_{AB} = (j - 1)(k-1)$	SS_{AB}/df_{AB}	MS_{AB}/MS_w
within	SS_W	$df_w = jk(n-1)$	SS_w/df_w	
total	SS_T	$df_t = n - 1$		

4. SPSS Results for Sample Data

SPSS results of the two-way ANOVA are provided below. The GENERAL LINEAR MODEL->UNIVARIATE command was used, and a model without interaction was specified. This will be illustrated during the chat.

Descriptive Statistics

Dependent Variable: achievement

instruction	sex	Mean	Std. Deviation	N
coop	f	72.0000	1.41421	2
	m	78.5000	.70711	2
	Total	75.2500	3.86221	4
lecture	f	87.5000	.70711	2
	m	90.0000	1.41421	2
	Total	88.7500	1.70783	4
self	f	81.5000	2.12132	2
	m	87.0000	1.41421	2
	Total	84.2500	3.50000	4
Total	f	80.3333	7.08990	6
	m	85.1667	5.41910	6
	Total	82.7500	6.52443	12

Tests of Between-Subjects Effects

Dependent Variable: achievement

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	448.083(a)	3	149.361	59.251	.000
Intercept	82170.750	1	82170.750	32596.661	.000
instruction	378.000	2	189.000	74.975	.000
sex	70.083	1	70.083	27.802	.001
Error	20.167	8	2.521		
Total	82639.000	12			
Corrected Total	468.250	11			

a R Squared = .957 (Adjusted R Squared = .941)

Estimates

Dependent Variable: achievement

instruction	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
coop	75.250	.794	73.419	77.081
lecture	88.750	.794	86.919	90.581
self	84.250	.794	82.419	86.081

Pairwise Comparisons

Dependent Variable: achievement

(I) instruction	(J) instruction	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
					Lower Bound	Upper Bound
coop	lecture	-13.500(*)	1.123	.000	-16.886	-10.114
	self	-9.000(*)	1.123	.000	-12.386	-5.614
lecture	coop	13.500(*)	1.123	.000	10.114	16.886
	self	4.500(*)	1.123	.012	1.114	7.886
self	coop	9.000(*)	1.123	.000	5.614	12.386
	lecture	-4.500(*)	1.123	.012	-7.886	-1.114

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.

Pairwise Comparisons

Dependent Variable: achievement

(I) sex	(J) sex	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
					Lower Bound	Upper Bound
f	m	-4.833(*)	.917	.001	-6.947	-2.719
m	f	4.833(*)	.917	.001	2.719	6.947

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.

5. APA Styled Presentation

Table 5
ANOVA Results and Descriptive Statistics for Achievement by Instructional Type and Sex

Variable	Mean	SD	n
Female			
Coop	72.00	1.41	2
Self	81.50	2.12	2
Lecture	87.50	0.71	2
Male			
Coop	78.50	0.71	2
Self	87.00	1.41	2
Lecture	90.00	1.41	2

Source	SS	df	MS	F
Instruction	378.00	2	189.00	74.98*
Sex	70.08	1	70.08	27.80*
Error	20.17	8	2.52	

Note: $R^2 = .96$, adj. $R^2 = .94$. Coop = co-operative learning, Self = self-paced, And Lecture = lecture instruction.
* $p < .05$

Table 6
Comparisons of Mean Differences in Achievement by Instruction

Comparison	Estimated Mean Difference	Standard Error of Difference	Bonferroni Adjusted 95% CI
Coop vs. Lecture	-13.50*	1.12	-16.89, -10.11
Coop vs. Self	-9.00*	1.12	-12.39, -5.61
Lecture vs. Self	4.50*	1.12	1.11, 7.89

Note: Coop = co-operative learning, Self = self-paced, And Lecture = lecture instruction.
* $p < .05$, where p-values are adjusted using the Bonferroni method.

ANOVA results show that achievement differs by both student sex and instructional type. Males demonstrated greater achievement (M = 85.17, SD = 5.42) than females (M = 80.33, SD = 7.09). Each pairwise comparison among instructional types is statistically significant. Among instructional types, students in lecture demonstrated the highest achievement, students in self-paced the next highest, and students in co-operative learning the lowest.

6. Regression Results

Multiple regression results for these data are provided below using the linear model listed above.

Descriptive Statistics

	Mean	Std. Deviation	N
achievement	82.7500	6.52443	12
lecture	.3333	.49237	12
self	.3333	.49237	12
male	.5000	.52223	12

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.978(a)	.957	.941	1.58771	.957	59.251	3	8	.000

a Predictors: (Constant), male, self, lecture

ANOVA(c)

Model			Sum of Squares	df	Mean Square	F	Sig.	R Square Change
1	Subset Tests	lecture, self	378.000	2	189.000	74.975	.000(a)	.807
		male	70.083	1	70.083	27.802	.001(a)	.150
	Regression		448.083	3	149.361	59.251	.000(b)	
	Residual		20.167	8	2.521			
	Total		468.250	11				

a Tested against the full model.
b Predictors in the Full Model: (Constant), male, self, lecture.
c Dependent Variable: achievement

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	72.833	.917		79.455	.000	70.719	74.947
	lecture	13.500	1.123	1.019	12.025	.000	10.911	16.089
	self	9.000	1.123	.679	8.017	.000	6.411	11.589
	male	4.833	.917	.387	5.273	.001	2.719	6.947

a Dependent Variable: achievement

7. Exercises

(a) Does student mathematics motivation vary by teacher and workbook? Students in each of three teacher's were randomly assigned to either mathematics workbook A, B, or C, so in each class each of the three workbooks was used by different students. Motivation scores derived from a scale that varies from 5 (low) to 15 (high).

Table 7a
Mathematics Motivation by Workbook and Teacher

Motivation	Workbook	Teacher
6	A	Smith
8	A	Smith
6	B	Smith
7	B	Smith
5	C	Smith
6	C	Smith
9	A	Griffin
10	A	Griffin
7	B	Griffin
9	B	Griffin
6	C	Griffin
8	C	Griffin
14	A	Marshall
12	A	Marshall
11	B	Marshall
12	B	Marshall
10	C	Marshall
11	C	Marshall

(b) Does student mathematics motivation vary by teacher, workbook, and student sex? Students in each of three teacher's were randomly assigned to either mathematics workbook A, B, or C, so in each class each of the three workbooks was used by different students. Motivation scores derived from a scale that varies from 5 (low) to 15 (high).

Table 7b
Mathematics Motivation by Workbook, Teacher, and Student Sex

Motivation	Workbook	Teacher	Student Sex
8	A	Smith	M
9	A	Smith	M
7	B	Smith	M
8	B	Smith	M
7	C	Smith	M
6	C	Smith	M
9	A	Griffin	M
10	A	Griffin	M
7	B	Griffin	M
9	B	Griffin	M
6	C	Griffin	M
8	C	Griffin	M
14	A	Marshall	M
12	A	Marshall	M
11	B	Marshall	M
12	B	Marshall	M
10	C	Marshall	M
11	C	Marshall	M
9	A	Smith	F
10	A	Smith	F
8	B	Smith	F
10	B	Smith	F
7	C	Smith	F
8	C	Smith	F
10	A	Griffin	F
11	A	Griffin	F
11	B	Griffin	F
12	B	Griffin	F
8	C	Griffin	F
10	C	Griffin	F
15	A	Marshall	F
13	A	Marshall	F
15	B	Marshall	F
13	B	Marshall	F
12	C	Marshall	F
13	C	Marshall	F

8. Exercise Answers

(a) Two-way ANOVA for Motivation

Data Plotted

https://spreadsheets.google.com/ccc?key=0AoKw33oyzB1NdDAzQTh2R2twOHRpdlJFRFM2eEM0MFE&hl=en&authkey=CKiCueAO

SPSS Commands:

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  /EMMEANS = TABLES(workbook) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(teacher) COMPARE ADJ(BONFERRONI)
  /PRINT = DESCRIPTIVE
  /CRITERIA = ALPHA(.05)
  /DESIGN = workbook teacher .
```

Descriptive Statistics

Dependent Variable: motivation

workbook	teacher	Mean	Std. Deviation	N
A	Griffin	9.5000	.70711	2
	Marshall	13.0000	1.41421	2
	Smith	7.0000	1.41421	2
	Total	9.8333	2.85774	6
B	Griffin	8.0000	1.41421	2
	Marshall	11.5000	.70711	2
	Smith	6.5000	.70711	2
	Total	8.6667	2.42212	6
C	Griffin	7.0000	1.41421	2
	Marshall	10.5000	.70711	2
	Smith	5.5000	.70711	2
	Total	7.6667	2.42212	6
Total	Griffin	8.1667	1.47196	6
	Marshall	11.6667	1.36626	6
	Smith	6.3333	1.03280	6
	Total	8.7222	2.58515	18

Tests of Between-Subjects Effects

Dependent Variable: motivation

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	102.222(a)	4	25.556	29.171	.000
Intercept	1369.389	1	1369.389	1563.107	.000
workbook	14.111	2	7.056	8.054	.005
teacher	88.111	2	44.056	50.288	.000
Error	11.389	13	.876		
Total	1483.000	18			
Corrected Total	113.611	17			

a R Squared = .900 (Adjusted R Squared = .869)

Estimates

Dependent Variable: motivation

workbook	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
A	9.833	.382	9.008	10.659
B	8.667	.382	7.841	9.492
C	7.667	.382	6.841	8.492

Pairwise Comparisons

Dependent Variable: motivation

(I) workbook	(J) workbook	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
					Lower Bound	Upper Bound
A	B	1.167	.540	.150	-.317	2.651
	C	2.167(*)	.540	.004	.683	3.651
B	A	-1.167	.540	.150	-2.651	.317
	C	1.000	.540	.261	-.484	2.484
C	A	-2.167(*)	.540	.004	-3.651	-.683
	B	-1.000	.540	.261	-2.484	.484

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.

Estimates

Dependent Variable: motivation

teacher	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Griffin	8.167	.382	7.341	8.992
Marshall	11.667	.382	10.841	12.492
Smith	6.333	.382	5.508	7.159

Pairwise Comparisons

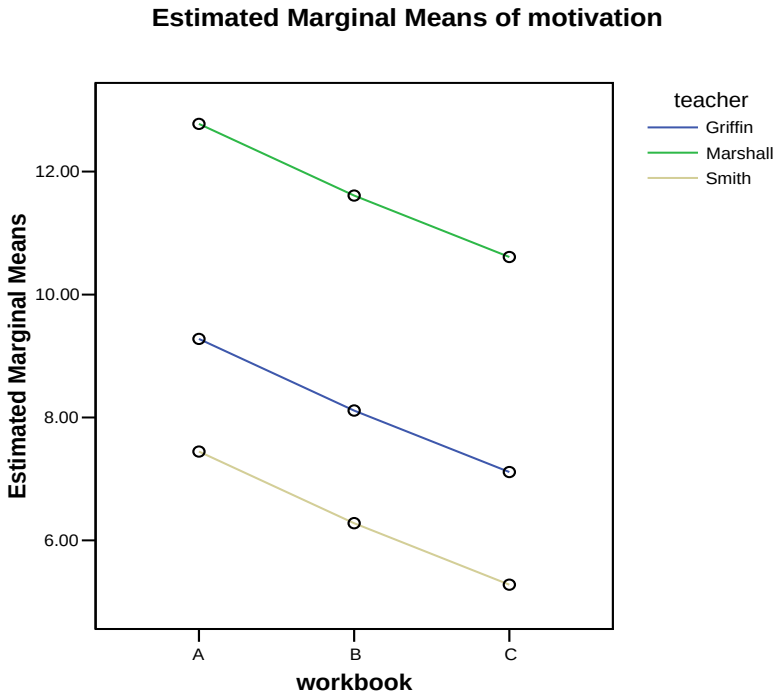
Dependent Variable: motivation

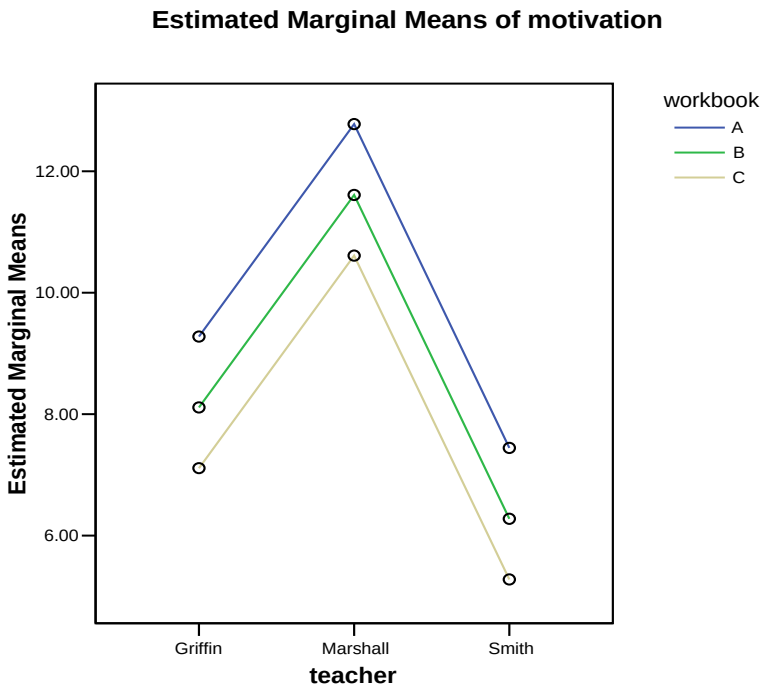
(I) teacher	(J) teacher	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
					Lower Bound	Upper Bound
Griffin	Marshall	-3.500(*)	.540	.000	-4.984	-2.016
	Smith	1.833(*)	.540	.014	.349	3.317
Marshall	Griffin	3.500(*)	.540	.000	2.016	4.984
	Smith	5.333(*)	.540	.000	3.849	6.817
Smith	Griffin	-1.833(*)	.540	.014	-3.317	-.349
	Marshall	-5.333(*)	.540	.000	-6.817	-3.849

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.





(b) Three-way ANOVA for Motivation

Data Plotted

https://spreadsheets.google.com/ccc?key=0AoKw33oyzB1NdF9QcmRLOUpSa1hvRVVfUzhFQWNaNFE&hl=en&authkey=CMbErdAO

SPSS Commands:

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  /INTERCEPT = INCLUDE
  /PLOT = PROFILE( workbook2*teacher2*sex2 sex2*teacher2*workbook2 workbook2*sex2*teacher2
teacher2*workbook2*sex2 )
  /EMMEANS = TABLES(OVERALL)
  /EMMEANS = TABLES(workbook2) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(teacher2) COMPARE ADJ(BONFERRONI)
  /EMMEANS = TABLES(sex2) COMPARE ADJ(BONFERRONI)
  /PRINT = DESCRIPTIVE
  /CRITERIA = ALPHA(.05)
  /DESIGN = workbook2 teacher2 sex2 .
```

Descriptive Statistics

workbook 2	teacher2	sex2	Mean	Std. Deviation	N
A	Griffin	F	10.5000	.70711	2
		M	9.5000	.70711	2
		Total	10.0000	.81650	4
	Marshall	F	14.0000	1.41421	2
		M	13.0000	1.41421	2
		Total	13.5000	1.29099	4
	Smith	F	9.5000	.70711	2
		M	8.5000	.70711	2
		Total	9.0000	.81650	4
	Total	F	11.3333	2.25093	6
		M	10.3333	2.25093	6
		Total	10.8333	2.20880	12
B	Griffin	F	11.5000	.70711	2
		M	8.0000	1.41421	2
		Total	9.7500	2.21736	4
	Marshall	F	14.0000	1.41421	2
		M	11.5000	.70711	2
		Total	12.7500	1.70783	4
	Smith	F	9.0000	1.41421	2
		M	7.5000	.70711	2
		Total	8.2500	1.25831	4
	Total	F	11.5000	2.42899	6
		M	9.0000	2.09762	6
		Total	10.2500	2.52713	12
C	Griffin	F	9.0000	1.41421	2
		M	7.0000	1.41421	2
		Total	8.0000	1.63299	4
	Marshall	F	12.5000	.70711	2
		M	10.5000	.70711	2
		Total	11.5000	1.29099	4
	Smith	F	7.5000	.70711	2
		M	6.5000	.70711	2
		Total	7.0000	.81650	4
	Total	F	9.6667	2.42212	6
		M	8.0000	2.09762	6
		Total	8.8333	2.32900	12
Total	Griffin	F	10.3333	1.36626	6
		M	8.1667	1.47196	6
		Total	9.2500	1.76455	12
	Marshall	F	13.5000	1.22474	6
		M	11.6667	1.36626	6
		Total	12.5833	1.56428	12
	Smith	F	8.6667	1.21106	6
		M	7.5000	1.04881	6
		Total	8.0833	1.24011	12
	Total	F	10.8333	2.38253	18
		M	9.1111	2.24628	18
		Total	9.9722	2.44349	36

Tests of Between-Subjects Effects

Dependent Variable: motiv2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	182.972(a)	5	36.594	42.224	.000
Intercept	3580.028	1	3580.028	4130.801	.000
workbook2	25.389	2	12.694	14.647	.000
teacher2	130.889	2	65.444	75.513	.000
sex2	26.694	1	26.694	30.801	.000
Error	26.000	30	.867		
Total	3789.000	36			
Corrected Total	208.972	35			

a R Squared = .876 (Adjusted R Squared = .855)

Estimates

Dependent Variable: motiv2

workbook2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
A	10.833	.269	10.284	11.382
B	10.250	.269	9.701	10.799
C	8.833	.269	8.284	9.382

Pairwise Comparisons

Dependent Variable: motiv2

(I) workbook2	(J) workbook2	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
					Lower Bound	Upper Bound
A	B	.583	.380	.406	-.380	1.547
	C	2.000(*)	.380	.000	1.036	2.964
B	A	-.583	.380	.406	-1.547	.380
	C	1.417(*)	.380	.002	.453	2.380
C	A	-2.000(*)	.380	.000	-2.964	-1.036
	B	-1.417(*)	.380	.002	-2.380	-.453

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a Adjustment for multiple comparisons: Bonferroni.

Estimates

Dependent Variable: motiv2

teacher2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Griffin	9.250	.269	8.701	9.799
Marshall	12.583	.269	12.034	13.132
Smith	8.083	.269	7.534	8.632

Pairwise Comparisons

Dependent Variable: motiv2

(I) teacher2	(J) teacher2	Mean Difference (I-J)	Std. Error	Sig.(a)	95% Confidence Interval for Difference(a)	
					Lower Bound	Upper Bound
Griffin	Marshall	-3.333(*)	.380	.000	-4.297	-2.370
	Smith	1.167(*)	.380	.014	.203	2.130
Marshall	Griffin	3.333(*)	.380	.000	2.370	4.297
	Smith	4.500(*)	.380	.000	3.536	5.464
Smith	Griffin	-1.167(*)	.380	.014	-2.130	-.203
	Marshall	-4.500(*)	.380	.000	-5.464	-3.536

Based on estimated marginal means

* The mean difference is significant at the .05 level.

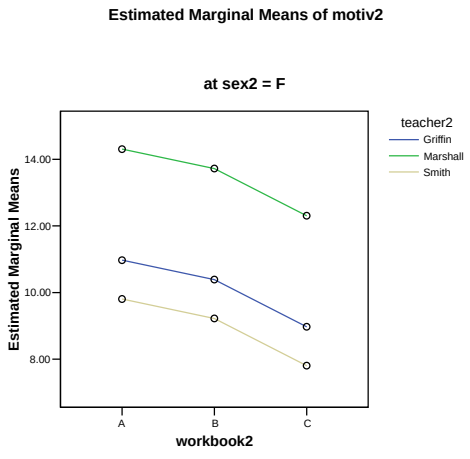
a Adjustment for multiple comparisons: Bonferroni.

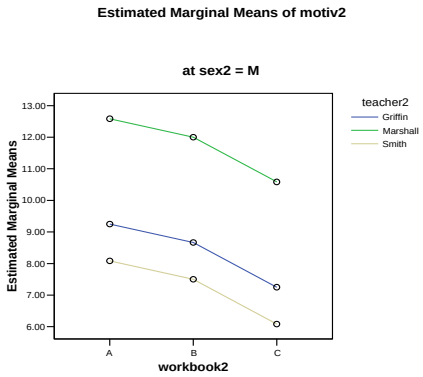
Estimates

Dependent Variable: motiv2

sex2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
F	10.833	.219	10.385	11.281
M	9.111	.219	8.663	9.559

Many graphs can be made with three IVs, but two examples are shown below.





APA styled Table

With a three-way ANOVA one must find a compact way to present cell means and ANOVA summary information. One approach is to present a table of means, SD, and n, then present ANOVA summary table separately.

Table x
Descriptive Statistics of Motivation by Instructor, Workbook, and Student Sex

	Instructor								
	Smith			Griffin			Marshall		
Workbook by Student Sex	M	SD	n	M	SD	n	M	SD	n
Males									
A	8.50	0.71	2	9.50	0.71	2	13.00	1.41	2
B	7.50	0.71	2	8.00	1.41	2	11.50	0.71	2
C	6.50	0.71	2	7.00	1.41	2	10.50	0.71	2
Females									
A	9.50	0.71	2	10.50	0.71	2	14.00	1.41	2
B	9.00	1.41	2	11.50	0.71	2	14.00	1.41	2
C	7.50	0.71	2	9.00	1.41	2	12.5	0.71	2

Note: N = 36, Grand M = 9.97, SD = 2.44

Table x
ANOVA Summary for Motivation by Instructor, Workbook, and Student Sex

Source	SS	df	MS	F
Instructor	130.89	2	65.44	75.51*
Workbook	25.39	2	12.69	14.65*
Student Sex	26.69	1	26.69	30.80*
Error	26.00	30	0.87	

Note: R^2 = .88, adj. R^2 = .86.
* $p < .05$

Follow these tables with tables of multiple comparisons where needed, e.g. for instructor and workbook.

Since motivation varies by sex, report marginal sex motivation means in text.