Multiple Linear Regression with Qualitative and Quantitative Independent Variables

1. Regression Equation

With both qualitative and quantitative predictors, the regression equation remains unchanged except with the addition of coefficients to capture the statistical effect of the predictors.

Table 1: Fictional Math Test Scores by Teacher, Student IQ, and Student Motivation

Math Test Scores	Teacher	IQ	Motivation	Smith	Collins	Brown
70.00	Brown	89.00	7.00	0	0	1
71.00	Brown	95.00	6.00	0	0	1
73.00	Brown	92.00	7.00	0	0	1
74.00	Brown	95.00	6.00	0	0	1
79.00	Smith	97.00	9.00	1	0	0
80.00	Smith	96.00	8.00	1	0	0
82.00	Smith	100.00	9.00	1	0	0
83.00	Smith	101.00	10.00	1	0	0
87.00	Collins	98.00	8.00	0	1	0
88.00	Collins	101.00	11.00	0	1	0
90.00	Collins	105.00	12.00	0	1	0
91.00	Collins	109.00	9.00	0	1	0

Data are available here:

http://www.bwgriffin.com/gsu/courses/edur8132/notes/Notes_8g_fictional_math_scores.sav

Using the fictional data in Table 1 above, assume we are interested in learning whether math test scores differ by type instrutor controlling for student IQ and motivation levels. The regression would be:

$$Y_i = b_0 + b_1 Smith_{1i} + b_2 Collins_{2i} + b_3 IQ_{3i} + b_4 MOTIVATION_{4i} + e_i, \quad (1)$$

where Smiht (1 = in Smith's class, 0 = other) and Collins (1 = in Collin's class, 0 = other) are dummy variables. The SPSS estimates are provided below in Table 2.

Table 2: SPSS results for data in Table 1

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	32.365	10.362		3.123	.017	7.862	56.867
	Smith	6.171	1.267	.408	4.869	.002	3.175	9.168
	Collins	12.023	1.721	.796	6.987	.000	7.954	16.092
	IQ	.415	.113	.306	3.667	.008	.147	.682
	Motivation	.177	.330	.045	.537	.608	-.603	.957

a Dependent Variable: Math_Scores

The sample prediction equation for these data is

$$Y' = 32.365 + 6.171(Smith) + 12.023(Collins) + 0.415(IQ) + 0.177(MOTIVATION) \quad (2)$$

Since regression equation contains multiple predictors, the represent partial statistical effects—the statistical association between X_1 and Y controlling for X_2. This is the same logic discussed earlier with multiple regressions. Since this above equation contains both qualitative and quantitative predictors, this model is identical to an Analysis of Covariance (ANCOVA) where the two quantitative predictors, IQ and Motivation, are known as *covariates*. These are control variables used to adjust or equate groups, or to partial effects of confounding variables.

Interpretation of coefficients remains the same as with previous multiple regression models discussed. One minor difference is that dummy variables now represented the *adjusted mean difference* between groups, adjusted for the statistical effects of the quantitative predictors (the covariates).

(1) What is the literal interpretation for b_0 = 32.365?
(2) What is the literal interpretation for b_1 = 6.171(Smith)?
(3) What is the literal interpretation for b_2 =12.023(Collins)?
(4) What is the literal interpretation for b_3 = 0.415(IQ)?
(5) What is the literal interpretation for b_4 =0.177(MOTIVATION)?

2. Predicted Values

The observed, unadjusted means for achievement, IQ, and motivation are presented in Table 3 below.

Table 3: Descriptive Statistics for Math Test Scores, IQ, and Motivation by Instructor and Overall

	Math Test Scores		IQ		Motivation		n
	M	SD	M	SD	M	SD	
Brown	72.00	1.825	92.75	2.872	6.50	0.577	4
Smith	81.00	1.825	98.50	2.380	9.00	0.817	4
Collins	89.00	1.825	103.25	4.787	10.00	1.825	4
Overall	80.667	7.4386	98.1667	5.491	8.5000	1.8829	12

A benefit of the inclusion of covariates, or quantitative predictors, when groups are compared is the statistical adjustment of group means and mean differences among groups. This statistical adjustment may provide some insight into how the groups may perform on the DV if each group scored the same on the covariates. This statistical adjustment is the result of partialing effects of regression.

As noted above, the prediction equation for this model is:

$$Y' = 32.365 + 6.171(\text{Smith}) + 12.023(\text{Collins}) + 0.415(\text{IQ}) + 0.177(\text{MOTIVATION}) \qquad (2)$$

To obtain predicted means, or adjusted means, one must substitute the mean value of the covariates into the regression equation. For the current example, these values would be used:

Mean of IQ = 98.1667
Mean of Motivation = 8.500

$$Y' = 32.365 + 6.171(\text{Smith}) + 12.023(\text{Collins}) + 0.415(\mathbf{IQ = 98.1667}) + 0.177(\mathbf{MOTIVATION = 8.50})$$

(1) What is the predicted mean (adjusted mean) for Brown's class?
(2) What is the predicted mean (adjusted mean) for Smith's class?
(3) What is the predicted mean (adjusted mean) for Collin's class?

Table 4: Observed Means and Adjusted Means

Instructor	Observed Mean	Adjusted Mean
Brown	72.00	
Smith	81.00	
Collins	89.00	

(4) How do the adjusted means (estimated means or predicted means) differ from the observed means? Why does this difference occur (partialing effect, handicapping)?

Linked spreadsheet below will calculated predicted means:

http://tinyurl.com/24x9p9k

https://spreadsheets.google.com/ccc?key=0AoKw33oyzB1NdDVuQUc5dlVuLTlyWFNWbDZDU082emc&hl=en &authkey=CJv_-_0P

3. Model Fit and Model Statistical Inference

The usual measures of fit and inference for the overall models continues to apply here.

4. Global Effects, ΔR^2, and the Partial F Test of ΔR^2

Statistical inference regarding the global effect, as measured by $\Delta R^2(X_k)$, continues to hold here. To illustrate, the overall statistical effect of instructor upon math scores will be tested. The reduced model contains only IQ and Motivation:

$$Y_i = b_0 + \qquad b_3 IQ_{3i} + b_4 MOTIVATION_{4i} + e_i, \qquad (3)$$

and the full model contains IQ, Motivation, and Instructor dummy variables:

$$Y_i = b_0 + b_1 Smith_{1i} + b_2 Collins_{2i} + b_3 IQ_{3i} + b_4 MOTIVATION_{4i} + e_i, \qquad (1)$$

Null hypothesis for the instructor statistical effect:

H_0: ΔR^2(instructor) = ΔR^2(Smith, Collins) = 0.00.

This hypothesis can be tested by hand or in SPSS.

Table 5

Model	R^2	Regression df	Error df
Y' = reduced model 3 above	.870	2	9
Y' = full model 4 above	.984	4	7
ΔR^2(Instructor) =	.984-.870 = .114	ΔR^2 df_1 = 4-2 = 2	ΔR^2 df_2 = 7 (smaller df)

If calculated by hand, th F ratio is calculated is

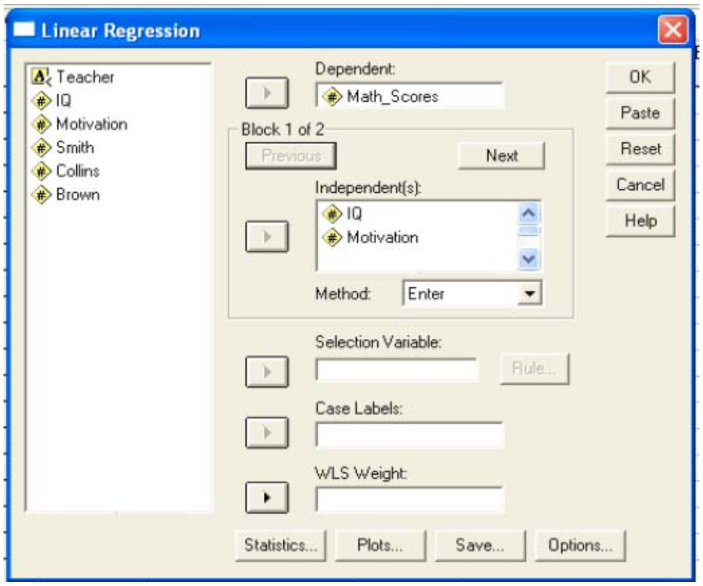
$$F = \frac{\Delta R^2 / (df_{2reduced} - df_{2full})}{(1 - R^2_{full}) / f_{2full}} = \frac{.114/(9-7)}{(1-.984)/7} = \frac{.057}{.0022857} = 24.937$$

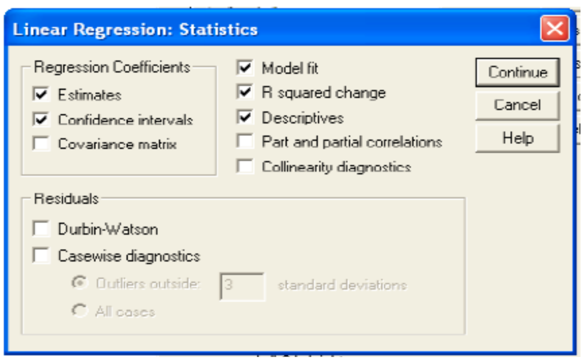
The df for this test are:

$df_1 = df_{2reduced} - df_{2full} = 9 - 7 = 2$, and
$df_2 = df_{2full} = 7$.

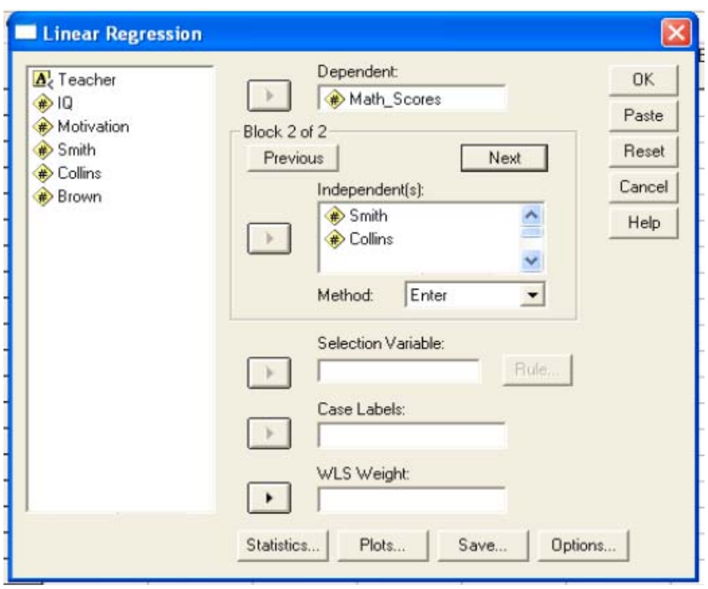
The critical F at $\alpha = .05$ would be 4.74. Since F = 24.937 is greater than critical F = 4.74, reject H_0 and conclude teachers do contribute to variability in math test scores in these data.

In SPSS:
1. Choose Regression, enter Math Scores in the Dependent box
2. Enter IQ and Motivation in Independents box, then click on Statistics->R-square Change->Continue





3. Click Next, then enter Smith and Collins dummy variables in IV box
4. Click Ok



See image below for SPSS results showing test of global effect ΔR^2(Instructor).

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.933[a]	.870	.841	2.96818	.870	30.044	2	9	.000
2	.992[b]	.984	.975	1.17188	.114	25.369	2	7	.001

a. Predictors: (Constant), Motivation, IQ

b. Predictors: (Constant), Motivation, IQ, Smith, Collins

$\Delta R2$(instructor) = .114

F ratio for test = 25.369

df 1 = 9-7 = 2

df 2 = 7

p-value for $\Delta R2$(instructor) = .001 (this is significant since p < .05 or .01, so reject Ho)

Recall that the hand-calculated F ratio was F = 24.937. The difference between this value and the value of 25.369 reported above is due to rounding error (level of precision with which R^2 is reported in SPSS)..

5. Inferential Procedures for Regression Coefficients

For variables that take one column, or vector, of data (such as the quantitative predictors), the t-ratio of b/se is sufficient for hypothesis testing. This is covered elsewhere and won't be repeated here.

6. Pairwise Comparisons Among IV Categories

For categorical (qualitative) predictors with more than two categories, such as instructor in the current example, one may need to perform pairwise comparisons to identify statistical difference if the Global Effect test is statistically significant, i.e., H_0: ΔR^2(instructor) = 0.00 is rejected.

Both Bonferroni and Scheffé can be used as before. One must perform comparisons among the adjusted mean differences, which are provided by the regression coefficients. With the current example regression equation:

$$Y_i = b_0 + b_1 Smith_{1i} + b_2 Collins_{2i} + b_3 IQ_{3i} + b_4 MOTIVATION_{4i} + e_i, \quad (1)$$

here

b_1 = adjusted mean difference in math scores between Smith's class and Brown's class;
b_2 = adjusted mean difference in math scores between Collin's class and Brown's class.

SPSS Results

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	32.365	10.362		3.123	.017	7.862	56.867
	IQ	.415	.113	.306	3.667	.008	.147	.682
	Motivation	.177	.330	.045	.537	.608	-.603	.957
	Smith	6.171	1.267	.408	4.869	.002	3.175	9.168
	Collins	12.023	1.721	.796	6.987	.000	7.954	16.092

a Dependent Variable: Math_Scores

b_1 = Smith vs. Brown = 6.171 (se = 1.267)
b_2 = Collin vs. Brown = 12.023 (se = 1.721)

The last comparison is between Smith and Collins, so re-run the regression and make Collins the reference group:

$Y_i = b_0 + b_1Smith_{1i} + b_2Brown_{2i} + b_3IQ_{3i} + b_4MOTIVATION_{4i} + e_i$, (4)

SPSS Results

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	44.388	11.598		3.827	.006	16.963	71.813
	IQ	.415	.113	.306	3.667	.008	.147	.682
	Motivation	.177	.330	.045	.537	.608	-.603	.957
	Smith	-5.852	1.012	-.387	-5.784	.001	-8.244	-3.460
	Brown	-12.023	1.721	-.796	-6.987	.000	-16.092	-7.954

a Dependent Variable: Math_Scores

b_1 = Smith vs. Collin = -5.852 (se = 1.012)

Standard errors for each of the regression coefficients are reported by SPSS, and calculation of the confidence intervals for the adjusted mean differences are performed as normal:

Bonferroni CI:
$b \pm se \times$ Bonferroni critical t

Scheffé:
$b \pm se \times$ Scheffé critical t

As illustration, use of the following spreadsheet to calculate CI will be used:

http://www.bwgriffin.com/gsu/courses/edur8132/notes/bonferroni_scheffe_ci.htm

The comparisons of interest are:

Table 6: Comparison Among Teachers

Comparison	Estimated Adjusted Mean Difference (b)	Standard Error of Difference (se b)
Smith vs. Brown	6.171	1.267
Collins vs. Brown	12.023	1.721
Smith vs. Collins	-5.852	1.012

Note: Adjusted mean differences and se taken from SPSS regression coefficients.

Other information needed from SPSS to calculate CI:

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	599.054	4	149.763	109.054	.000(a)
	Residual	9.613	7	1.373		
	Total	608.667	11			

a Predictors: (Constant), Brown, Smith, Motivation, IQ
b Dependent Variable: Math_Scores

With this information, the spreadsheet provides the following calculations for the CI:

Table 7: Spreadsheet CI

Comparison	Adjusted mean difference	se	Bonferroni LL	Bonferroni UL	Scheffé LL	Scheffé UL
Smith vs. Brown	6.1710	1.2670	2.2084	10.1336	2.2710	10.0710
Collins vs. Brown	12.0230	1.7210	6.6405	17.4055	6.7256	17.3204
Smith vs. Collins	-5.8520	1.0120	-9.0171	-2.6869	-8.9671	-2.7369

7. APA Style Results

Table 8
Descriptive Statistics and Correlations Among Math Scores, Teachers, IQ, and Motivation

Variable	Correlations				
	Math Scores	IQ	Motivation	Smith	Collins
Math Scores	---				
IQ	.90*	---			
Motivation	.81*	.71*	---		
Smith	.03	.05	.20	---	
Collins	.83*	.68*	.59*	-.50	---
Mean	80.67	98.17	8.50	0.33	0.33
SD	7.44	5.49	1.88	0.49	0.49

Note. Smith (1 = students in Smith's class, 0 = others) and Collins (1 = students in Collin's class, 0 = others) are dummy variables; n = 12.
*p<.05.

Table 9
Regression of Math Scores on Teachers, IQ, and Motivation

Variable	b	se	ΔR^2	95%CI	F	t
IQ	0.42	0.11	.03	0.15, 0.68		3.67*
Motivation	0.18	0.33	.01	-0.60, 0.96		0.54
Teacher			.11		25.37*	
Smith	6.17	1.27		3.18, 9.17		4.87*
Collins	12.02	1.72		7.95, 16.09		6.99*
Intercept	32.37	10.36		7.86, 56.87		3.12*

Note. R^2 = .98, adj. R^2 = .98, $F_{4,7}$ = 109.05*, MSE = 1.373, n = 12. ΔR^2 represents the semi-partial multiple correlation or the increment in R^2 due to adding the respective variable. Smith (1 = students in Smith's class, 0 = others) and Collins (1 = students in Collin's class, 0 = others) are dummy variables.
*p<.05.

Table 10
Comparisons of Adjusted Mean Math Scores Among Instructors

Comparisons	Estimated Adjusted Mean Difference	Standard Error of Difference	Bonferroni Adjusted .95CI
Smith vs. Brown	6.17*	1.27	2.21, 10.13
Collins vs. Brown	12.02*	1.72	6.64, 17.41
Smith vs. Collins	-5.85*	1.01	-9.02, -2.69

Note. Math score comparisons adjusted based upon IQ and Motivation.
*p<.05, where p-values are adjusted using the Bonferroni method.

Intelligence is positively related to math scores, and there are statistical differences in mean math scores among instructors. Motivation, once instructor and IQ are controlled, does not appear to be related to math scores. All pairwise comparisons were performed and all were statistically significant at the 5% level using the Bonferroni adjustment. Students in Collins' class performed best, those in Smith's performed worst, those in Brown's class scored between Collins' and Smith's classes.

8. Exercises

(1) According to the leadership literature, there are a number of different leadership styles. Listed below are scores obtained from an instrument designed to measure a particular leadership style, which will be referred to as style X. Of interest is whether X differs by school district type in terms of urbanity, and by years of experience. A stratified random sample of school principals were selected from three district types (mostly urban, mostly suburban, and mostly rural).

The scores on style X range from 100 to 0. The closer the score to 100, the more the respondent conforms to style X, while the closer the score to 0, the less the respondent conforms to style X.

Is there any evidence that X differs among the three district types, once years of experience is taken into account?

Years of Experience	District Type	Style X
21	urban	85
20	urban	98
18	urban	75
10	urban	63
17	urban	91
5	urban	49
4	urban	62
7	suburban	49
1	suburban	48
9	suburban	56
10	suburban	78
2	suburban	35
4	suburban	50
1	rural	33
16	rural	95
3	rural	26
2	rural	11
2	rural	33
8	rural	25
14	rural	65

(2) A researcher is interested in learning whether frequency of reading at home to elementary-aged children produces differential effects on reading achievement. After obtaining information from a randomly selected sample of parents about this behavior, the following classifications and standardized achievement scores were recorded. (Note: frequency classifications as follows: a = less than once per month, b = once to three times per month, c = more than three times per month.) In addition to reading frequency, information regarding the family's status concerning whether or not the family's child receives either free or reduced lunch is recorded as a proxy for SES. Also, students' IQ scores were recorded.

IQ	SES	Freq. of Reading	Achievement
100	4	a	48
101	3	a	37
98	3	a	47
105	5	a	65
86	4	b	57
92	3	b	39
117	5	b	49
99	2	b	45
105	5	c	61
93	3	c	55
101	5	c	51
103	1	c	30

Note. SES scores range from al a low of 1 to a high of 5.

Is frequency of reading at home related to student reading achievement once SES and IQ are taken into account?

(3) An administrator wishes to know whether student behavioral problems can be linked to student performance. If students were suspended or reprimanded more than once, they are classified as having behavioral problems. In addition, each student's SES is known, and should be taken into account. The administrator randomly selects 13 students and collects the appropriate data.

Student	GPA	Student SES	Behavioral Problems
Bill	3.33	5	n
Bob	1.79	1	y
Stewart	2.21	4	n
Linda	3.54	5	y
Lisa	2.89	4	n
Ann	2.54	3	n
Fred	2.66	5	y
Carter	1.10	1	y
Bill	3.10	4	n
Sue	2.10	2	y
Kara	2.07	2	y
Loser	2.31	3	n
Kathy	3.67	4	n

Note. SES scores range from a low of 1 to a high of 5.

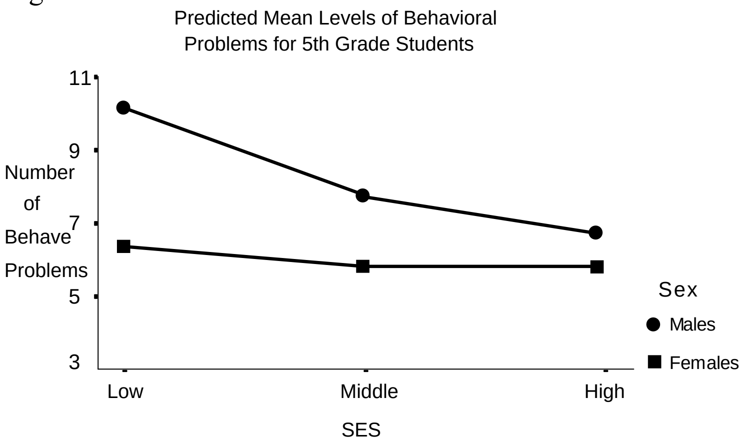
9. Answers to be added.

10. Interactions

This section introduces, very briefly, interactions. This will be covered in more detail in ANOVA models.

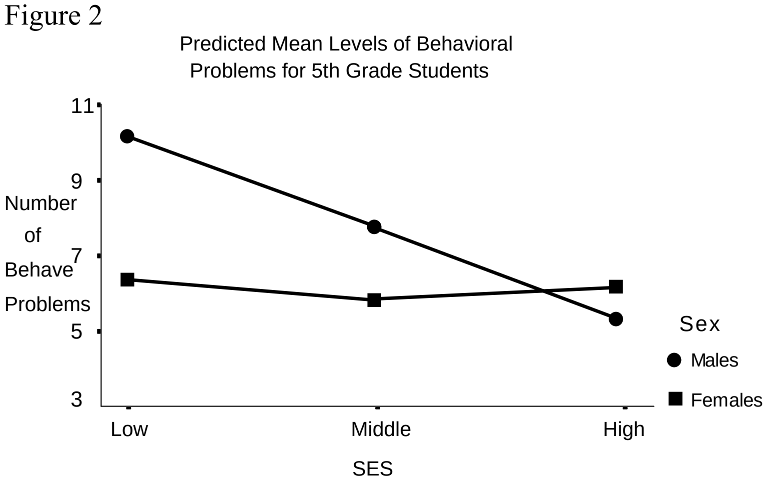
An interaction occurs when an independent variable's statistical effects (or differences) upon the dependent variable vary or differ across levels of a second independent variable. For example, if one were interested in examining the relationship between SES, sex, and the number of behavioral problems displayed among 5th grade students, one may find a pattern such as the one depicted in Figure 1.

Figure 1



Ordinal interaction in Figure 1 – note differences between sexes changes over levels of SES.

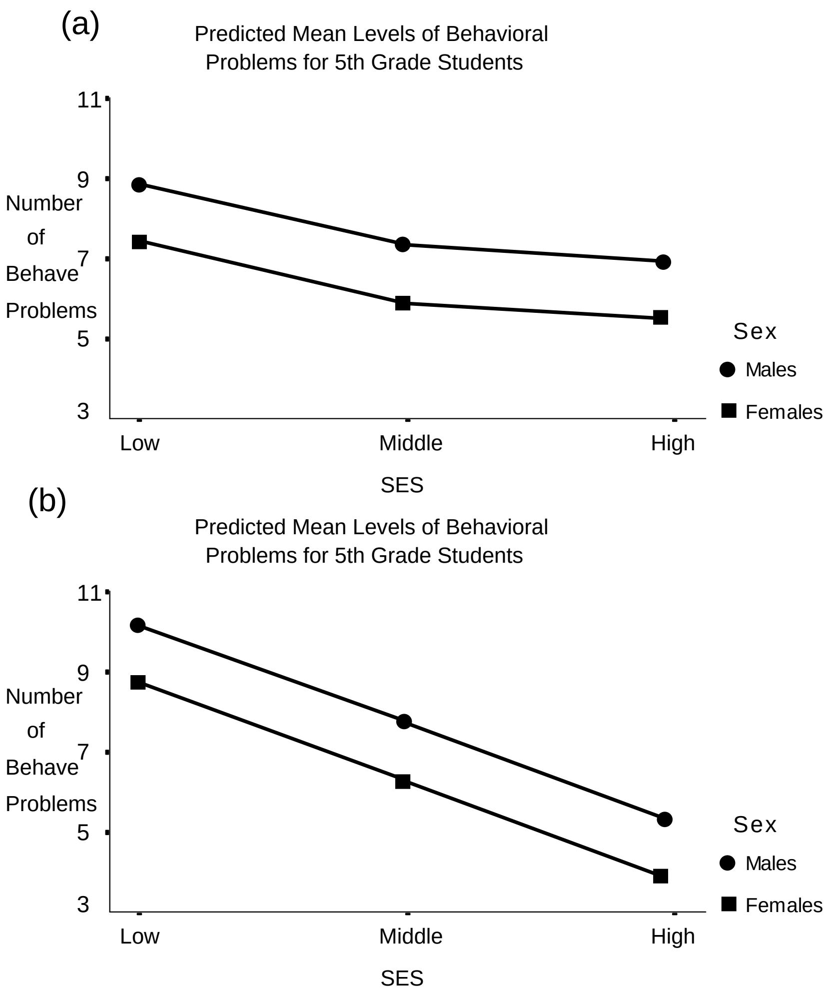
Figure 2



Disordinal interaction in Figure 2 – again note changes in Sex differences across levels of SES

Figures 3 (a) and (b) show data without presence of interaction – note Sex differences remain relatively consistent across levels of SES.

Figure 3



11. Interactions: Regression Equation

Table 11 below contains fictional data that demonstrates and interaction between Sex and SES.

Model without interaction:

$Y' = b_0 + b_1MALE + b_2HIGH + b_3LOW$

Interactions are model in regression via multiplicative terms. For two independent variables, X_1 and X_2, the interaction is found by including in the regression model the following product:

interaction = $X_1 * X_2$.

When one or more of the independent variables is categorical, each dummy representing the variable must be multiplied. For the current example, there will be two multiplicative terms to include in the regression model:

INT_1 = MALE * HIGH,

and

INT_2 = MALE * LOW.

Entering these terms into the sample regression model results in the following equations:

$$Y' = b_0 + b_1MALE + b_2HIGH + b_3LOW + b_4INT_1 + b_5INT_2 \quad (2)$$

or

$$Y' = b_0 + b_1MALE + b_2HIGH + b_3LOW + b_4(MALE*HIGH) + b_5(MALE*LOW). \quad (3)$$

Table 11
Fictional Data Demonstrating Interaction

behave	sex	ses	MALE	HIGH	LOW	MALE*HIGH	MALE*LOW
9	m	h	1	1	0	1	0
11	m	h	1	1	0	1	0
12	m	h	1	1	0	1	0
10	m	h	1	1	0	1	0
4	f	h	0	1	0	0	0
5	f	h	0	1	0	0	0
6	f	h	0	1	0	0	0
4	f	h	0	1	0	0	0
5	m	m	1	0	0	0	0
6	m	m	1	0	0	0	0
4	m	m	1	0	0	0	0
6	m	m	1	0	0	0	0
2	f	m	0	0	0	0	0
4	f	m	0	0	0	0	0
3	f	m	0	0	0	0	0
3	f	m	0	0	0	0	0
6	m	l	1	0	1	0	1
7	m	l	1	0	1	0	1
5	m	l	1	0	1	0	1
5	m	l	1	0	1	0	1
3	f	l	0	0	1	0	0
2	f	l	0	0	1	0	0
1	f	l	0	0	1	0	0
3	f	l	0	0	1	0	0

12. Interactions: Hypothesis Testing

As usual, the first hypothesis to test is that of the overall model: H_0: $R^2 = 0.00$. If this is rejected, the next hypothesis to test is that of the interaction effect, i.e., H_0: ΔR^2(interaction) = 0.00 or H_0: ΔR^2(MALE*HIGH, MALE*LOW) = 0.00. This will be illustrated for the sample data.

For the sample data, the reduced model R^2 is .836 with $df_2 = 20$. The full model R^2 is .9035 with $df_2 = 18$, so

$$\Delta R^2 = .9035 - .836 = 0.0675.$$

The partial F ratio is

$$F = \frac{\Delta R^2(X_k)/(df_{2r} - df_{2f})}{(1-R_f^2)/df_{2f}} = \frac{.0675/(20-18)}{(1 - .9035)/18} = \frac{0.0337}{0.0054} = 6.24.$$

which is statistically significant at the .05 level.

When an interaction is statistically significant, one need not perform significance testing for the independent variables associated with the interaction, i.e., one need not compute ΔR^2 and partial F tests for independent variables associated with the interaction.

13. Interactions: Regression Coefficient Interpretation

With interactions, one cannot simply interpret main effects; that is, one cannot simply rely upon the regression coefficients produced for the two independent variables. When interpreting the results, one must take into account the interaction. For example, consider interpretations for the current sample data. The regression results are

Y' = 3.00 + 2.25(MALE) + 1.75(HIGH) + -0.75(LOW) + 3.50(MALE*HIGH) + 1.25(MALE*LOW).

One might be tempted to claim that the estimated difference between males and females is 2.25 since this is the main effect of sex. However, since there are interactions present between sex and SES, one must examine the *simple main effects* for each variable.

What is meant by simple main effects? This means that one must interpret the effects of sex for each level or category of the variable in which sex interacts. That is, indicate whether males or females differ, and by how much, for each unique level of SES. Similarly, for each sex, indicate how the SES levels differ.

The easiest way to show these effects is to calculate the predicted means for each cell or combination of variables and to plot these on a graph like Figures 1 through 3. For example, the predicted means for males and females for each level of SES can be found using the regression equation, as demonstrated below:

Y' = 3.00 + 2.25(MALE) + 1.75(HIGH) + -0.75(LOW) + 3.50(MALE*HIGH) + 1.25(MALE*LOW).

SES = LOW and SEX = MALE
Y' = 3.00 + 2.25(1) + 1.75(0) + -0.75(1) + 3.50(0) + 1.25(1).
Y' = 3.00 + 2.25 - 0.75 + 1.25 = 5.75.

SES = LOW and SEX = FEMALE
Y' = 3.00 + 2.25(0) + 1.75(0) + -0.75(1) + 3.50(0) + 1.25(0).
Y' = 3.00 - 0.75 = 2.25.

SES = MIDDLE and SEX = MALE
Y' = 3.00 + 2.25(1) + 1.75(0) + -0.75(0) + 3.50(0) + 1.25(0).
Y' = 3.00 + 2.25 = 5.25.

SES = MIDDLE and SEX = FEMALE
Y' = 3.00 + 2.25(0) + 1.75(0) + -0.75(0) + 3.50(0) + 1.25(0).
Y' = 3.00 = 3.00.

SES = HIGH and SEX = MALE
Y' = 3.00 + 2.25(1) + 1.75(1) + -0.75(0) + 3.50(1) + 1.25(0).
Y' = 3.00 + 2.25 + 1.75 + 3.50 = 10.5.

SES = HIGH and SEX = FEMALE
Y' = 3.00 + 2.25(0) + 1.75(1) + -0.75(0) + 3.50(0) + 1.25(0).
Y' = 3.00 + 1.75 = 4.75.

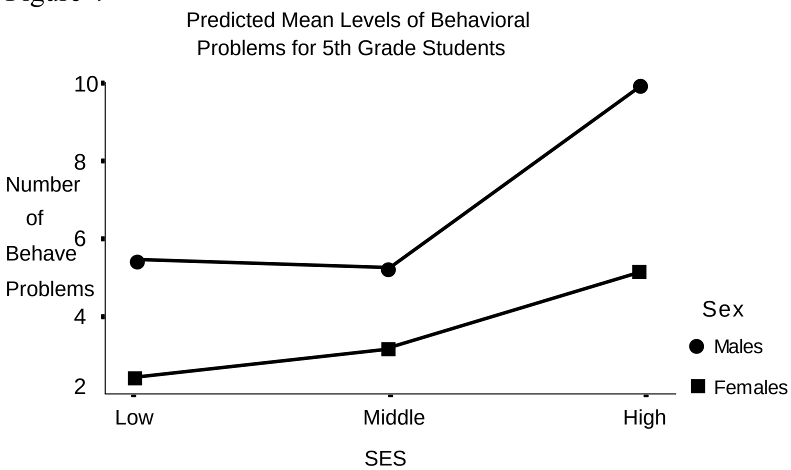
So the predicted means are:

Table 7

SEX	Low	SES Middle	High
Male	5.75	5.25	10.5
Female	2.25	3.00	4.75

which can be better illustrated via a graph, such as Figure 4.

Figure 4



Now the only thing left to do test the statistical significance of the various comparisons. This is easily accomplished using the computer procedure to calculate standard errors. For example, the difference between males and females, when SES is held constant at the middle level is $b_1 = 2.25$, which has a standard error of 0.707. In Table 2 the difference between males and females for each level of SES is calculated and reported.

Table 8

SEX	Low	SES Middle	High
Male	5.75	5.25	10.5
Female	2.25	3.00	4.75
Sex Difference	3.50	2.25	5.75
(standard error)	(0.707)	(0.707)	(0.707)

To find the estimated difference between males and females for low levels of SES, simply re-specify the regression model so that low SES represents the omitted reference group. When doing this, be careful that the proper interaction terms are also included (in this case, no interactions based upon the LOW dummy should be included). For example, to find the estimated difference between the sexes for low levels of SES, the revised regression equation is

$Y' = b_0 + b_1MALE + b_2HIGH + b_3MIDDLE + b_4(MALE*HIGH) + b_5(MALE*MIDDLE)$.

The resulting statistics are:

Y' = 2.25 + 3.50(MALE) + 2.50(HIGH) + 0.75(MIDDLE) + 2.25(MALE*HIGH) + -1.25(MALE*MIDDLE),

and the standard error for the difference between men and women is 0.707. To find the estimated difference and standard error for sex at high levels of SES, the following equation should be used:

$$Y' = b_0 + b_1MALE + b_2MIDDLE + b_3LOW + b_4(MALE*MIDDLE) + b_5(MALE*LOW). \quad (5)$$

The resulting statistics are:

$$Y' = 4.75 + 5.75(MALE) -1.75(MIDDLE) -2.50(LOW) -3.50(MALE*MIDDLE) -2.25(MALE*LOW).$$

One may also compare the three levels of SES for each distinct sex category using a method similar to the one just described. For this set of comparisons, either Scheffé or Bonferroni should be used for each comparison set (i.e., each set of comparisons per sex category).

Table 9
Descriptive Statistics and Correlations Among Behavioral Problems, SES, Sex, and the SES×Sex Interaction

Variable	Behave	Correlations High	Low	Male	Male * Low	Male * High
Behave	---					
High	.602*	---				
Low	-.317	-.500*	---			
Male	.688*	.000	.000	---		
Male * Low	.080	-.316	.632*	.447*	---	
Male * High	.842*	.632*	-.316	.447*	-.200	---
Mean	5.25	.33	.33	.50	.17	.17
SD	2.85	.48	.48	.51	.38	.38

Note. Male is a dummy variable (1 = male, 0 = female), as are High (1 = high SES, 0 = otherwise) and Low (1 = low SES, 0 = otherwise); n = 24.
*p<.05.

Table 10
Regression of Behavioral Problems on Sex and SES

Variable	b	se	ΔR^2	95%CI	F	t
Male	2.25	0.71	.05	0.76, 3.74	10.13*	3.18*
SES			.07		6.58*	
Low	-0.75	0.71		-2.24, 0.74		-1.06
High	1.75	0.71		0.26, 3.24		2.48*
Sex × SES			.07		6.29*	
Male × High	3.50	1.00		1.40, 5.60		3.50*
Male × Low	1.25	1.00		-0.85, 3.35		1.25
Intercept						

Note. R^2 = .90, adj. R^2 = .88, $F_{5,18}$ = 33.70*, MSE = 1.00, n = 24. ΔR^2 represents the semi-partial multiple correlation or the increment in R^2 due to adding the respective variable; Male is a dummy variable (1 = male, 0 = female), as are High (1 = high SES, 0 = otherwise) and Low (1 = low SES, 0 = otherwise).
*p < .05.

Table 5
Simple Main Effect Comparisons of Behavioral Problems by Sex Across Levels of SES

Contrast	Male vs. Female Mean Difference	Standard Error of Difference	Bonferroni Adjusted .95CI
SES Level			
Low	3.50*	0.71	1.64, 5.36
Middle	2.25*	0.71	0.39, 4.11
High	5.75*	0.71	3.89, 7.61

*p<.05, where p-values are adjusted using the Bonferroni method.

[Note:
1. Instead of sex pairwise comparisons for each level of SES in Table 5, one could also calculate and display SES comparisons separately for males and for females
2. The Bonferroni critical t used was 2.631 (df = 18, comparisons = 3).]

An interactive graphical of these results can be seen here:
http://tinyurl.com/2vk7l4s

Also here:
https://spreadsheets.google.com/ccc?key=0ArHM99WFArnmdGpYRzlRQ3FHQ09NWVVqcEdWOWhpcGc&hl=en&authkey=CMeshLcC

Means Per Cell Combination

	Male	Female
High	10.5	4.75
Middle	5.25	3
Low	5.75	2.25

Figure Showing Interaction: Sex Differences Across Levels of SES

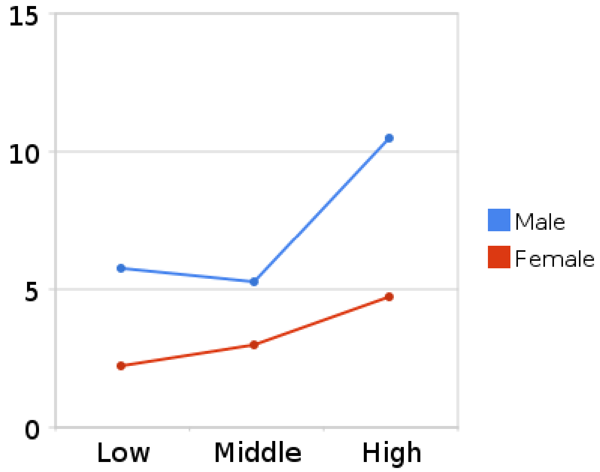


Figure Showing Same Data and Interaction: SES Differences Across Levels of Sex