Multiple Linear Regression: Two Quantitative IVs

The extension of simple linear regression to regression with two quantitative variables is straight-forward and requires the learning of only a few new concepts. As with simple regression, the role of multiple regression is twofold: explanation and prediction. The discussion of these two topics presented earlier continue to hold here.

The Regression Equation

Suppose a researcher is interested in determining whether academic achievement is related to students' time spent studying and their academic ability. Hypothetical data for these variables are presented in Table 1. In the corresponding regression equation for this model, achievement is denoted Y, time spent studying X_1, and academic ability X_2. The population regression model is

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_i, \quad (1)$$

where

Y_i signifies the i^{th} student's achievement score;
β_1 is the population partial regression coefficient expressing the relationship between X_1 and Y, controlling for X_2;
β_2 is the population partial regression coefficient expressing the relationship between X_2 and Y, controlling for X_1;
β_0 is the population intercept for the equation; and
ε_i is, supposedly, a random error.

Note the change in the interpretation of the regression coefficients. Now each β_i represents the partial effect of X_i on Y, controlling (or partially out) the effects of the other Xs. What is a partial effect? Often researchers wish to investigate models in which more than one IV can be used to explain Y. When this occurs, typically the Xs will be inter-related; that is, the Xs will be correlated. With the current example, both ability and time spent studying will likely be related to achievement. The question of interest is whether, for instance, time spent studying will affect achievement once ability is taken into account—i.e., controlled or partial out. Thus, multiple regression allows one to examine the effects of a given X upon Y while simultaneously taking into account the effects of other Xs. And this ability to partial-out the effects of Xs is the strength of multiple regression.

The multiple regression equation can easily be extended to any number of Xs. For example, one may want to model achievement using not only time spent studying and academic ability, but also learning style, prior exposure to the topic, parental support, instructional strategy used, etc. The regression equation is simply extended with additional regression coefficients and their respective Xs. For example,

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + ... + \beta_k X_k + \varepsilon_i, \quad (1)$$

where

β_i indicates the k^{th} coefficient for the k^{th} independent variable, X_k.

To keep things simple, the discussion of multiple regression will focus upon the two independent variable situation. The sample regression equation for the hypothetical example of achievement is:

$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + e_i, \quad (2)$$

where b_0 is the sample intercept; b_1 is the sample regression coefficient for X_1 controlling for the effect of X_2; b_2 is the sample regression coefficient for X_2 controlling for the effect of X_1; and e_i is the sample error term. Sample data for this example is given in Table 1.

The goal of regression is to find a mathematical solution for b_0, b_1, and b_2 that will best fit the data reflected in Y, X_1, and X_2. As before, OLS estimates will minimize the sum of the squared residuals, $\sum e_i^2$, and will therefore provide the best mathematical fit of the data.

Table 1
Achievement, Time Spent Studying, and Academic Ability

Achievement	Time (in hours)	Ability
88	8	6
75	6	2
64	0	2
99	9	9
95	5	9
93	8	7
85	7	5
82	5	4
79	1	5
78	1	3
91	4	7
85	4	9

Note. Higher scores indicate higher levels of each variable.

The Prediction Equation and Residuals

The prediction equation for the example data is:

$$Y' = b_0 + b_1X_1 + b_1X_2. \quad (3)$$

Actual OLS estimates for this model are:

$$Y' = 63.90 + 1.30(X_1) + 2.52(X_2).$$

Residuals are obtained in a manner identical to that described earlier. Namely, one obtains the predicted value Y' and subtracts this value from the observed Y, i.e.,

$$e_i = Y - Y'.$$

Consider, for instance, the residual for the first data entry in Table 1. The predicted value of achievement is:

$$\begin{aligned} Y' &= 63.90 + 1.30(8) + 2.52(6), \\ &= 63.90 + 1.30(8) + 2.52(6), \\ &= 63.90 + 10.4 + 15.12, \\ &= 89.42. \end{aligned}$$

The observed value of achievement is 88, so the residual is:

e_1 = Y - Y',
= 88 - 89.42,
= -1.42.

This negative value indicates that this person's score was over-predicted; that is, this person's observed score was less than the score predicted for this person given this individual's level of ability and time spent studying.

As previously noted, the goal of OLS is to obtain estimates for the regression coefficients that will provide the smallest possible residuals for all observations in the sample, and when certain assumptions are met, OLS estimates do provide the smallest possible residuals (for proof, see the Pedhazur text). In short, OLS attempts to find the regression line that passes through all observations and that provides the smallest set of squared residuals, $\sum e_i^2$.

Regression Coefficient Interpretation

The model intercept, b_0, is simply the point at which the regression line passes through the Y axis when *both* X_1 and X_2 equal zero. The other regression coefficients indicate the nature (direction and degree) of the *partial* relationship between an independent and dependent variable while controlling for other independent variables in the model. For example, b_1 in the achievement model equals 1.30. The partial effect indicates that a one unit increase in X_1 changes Y by b_1 (or 1.30) units, controlling for the other Xs. In terms of the example data, a one hour increase in the time spent studying is expected to increase achievement by 1.3 points, controlling for the effects of ability. Similar, since $b_2 = 2.52$, once could state that a one unit increase in ability results in an average increase of 2.52 points in achievement, controlling for the effects of study time.

Emphasis has been placed on the notion of partial effect. This implies that if one did not control for, say, ability, then the relationship found between time spent studying and achievement might be different. This fact can easily be illustrated. Suppose one estimated the relationship between time spent studying and achievement as follows:

$$Y' = b_0 + b_1X_1. \quad (4)$$

The sample estimates for this model are:

$$Y' = 73.17 + 2.34\ X_1.$$

Note that when one does not take into account the impact upon achievement of academic ability, time spent studying appears to have almost twice the estimated effect as found in the multiple regression model. That is, with the single regression model, a one hour increase in time spent studying is estimated to increase achievement by 2.34 points, but when ability is included in the model, the effect of one additional hour of study time on achievement is only an increase of 1.30 points. As this example illustrates, when multiple Xs are theoretically linked to the modeled Y, it is important that these Xs be included in the model in order to obtain the best estimates of the true relationships among the variables.

Another, and perhaps more informative, way to present the modeled effects of a given X is to indicate the change in Y associated with a given amount of change in X. For example, suppose one is most interested in learning by how much Y is likely to change if X changes by, say, five units. With the example data, one may be curious to know the amount of change in achievement that would be associated with an increase of five more hours spent studying during the week. To find this estimated change in Y, simply multiple the partial regression coefficient for X by the number of units of change in X. So if one were to increase the number of hours spent studying during the week by five, then achievement would be anticipated to increase by (1.30 * 5 = 6.5) 6.5 points, or perhaps over half a letter grade.

Overall Model Fit and Statistical Inference

As previously explained, model fit refers to the degree to which Y' approximates Y. Model fit, of the lack thereof, can be measured by the amount of variation in the residuals. The smaller this variation, the better the model reproduces the observed data, i.e., the better Y' estimates Y. Recall that residual variation is estimated via the mean square error, *MSE*, and the standard error of estimate, *SEE* ($\hat{\sigma}$). *MSE* is the variance error of the residuals, and *SEE* is the standard error of the residuals. In short, the smaller the value of MSE or SEE relative to the observed variance of Y or standard deviation of Y, respectively, the better the model fits or replicates the data.

Since both *MSE* and *SEE* are scale dependent, it may be difficult to interpret these indices to assess model fit. Scale free (or scale invariant) measures of model fit are Multiple R, R^2, and adjusted R^2. Multiple R is simply the Pearson correlation between Y and Y', and when this value is squared, the resultant index, R^2, indicates the proportion of variance in Y that can be explained or accounted for by the combination of Xs in the multiple regression model. Since OLS tends to maximize the estimated relationships among the Xs and Y, R^2 tends to overestimate the fit of the model to the data, so a better index of fit is adjusted R^2. For a more thorough treatment of these indices of model fit, see the earlier discussion presented in "Simple Linear Regression: One Quantitative IV."

In multiple regression, one should initially test the tenability of H_0: $R^2 = 0.00$ before proceeding to examine the individual regression coefficients. Should H_0 not be rejected, then one may tentatively conclude that estimated model does not adequate reproduce or explain the observed data, and therefore examination of the individual regression coefficients might be potentially misleading. As with simple linear regression, H_0 is tested via the overall F test. The F ratio is defined as

$$F = \frac{SSR/df_r}{SSE/df_e} = \frac{SSR/k}{SSE/(n - k - 1)} = \frac{MS_{reg}}{MSE} = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

where
SSR = regression sums of squares;
SSE = residual sums of squares;
df_r = regression degrees of freedom (also denoted df_1);
df_e = residual degrees of freedom (also denoted df_2);
k = number of independent variables (or vectors) in the model;
n = sample size (or number of observations in sample);
MS_{reg} = mean square (same as ANOVA) due to regression (e.g., between);
MSE = mean square error (same as ANOVA mean square within).

The observed R^2 for the achievement data is .874. Using the R^2 formula, the calculated overall F ratio is

$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} = \frac{.8742/2}{(1 - .8742)/(12 - 2 - 1)} = 31.27,$$

or

$$F = \frac{SSR/df_r}{SSE/df_e} = \frac{906.559/2}{130.441/9} = 31.27.$$

With $df_1 = k = 2$ and $df_2 = n - k - 1 = 12 - 2 - 1 = 9$, the .05 level critical F value is

$$_{.05}F_{2,9} = 4.26.$$

Since 31.27 is larger than 4.26, H_0 is rejected and one may conclude that the model of Y using X_1 and X_2 explains statistically more variability than would be expected by chance alone.

Of course p-values may also be provided for the F test, with the usual decision rule. The p-value for the obtained F ratio is .000, which is clearly less than .05, so the same conclude regarding H_0: $R^2 = 0.00$ is research.

ΔR^2, Semi-partial Correlation, and the Partial F Test of ΔR^2

Multiple regression provides estimates of partial effects for each X. Recall that partial effects represent the relationship between Y and a given X while other Xs are partialed-out or controlled. For some variables, particularly categorical variables, it may be necessary to perform significance tests of the partial effects by hand. Hypothesis testing of partial effects is most often performed using the partial F test. Hypotheses using the partial F test can be quite cumbersome, so before turning to some of the more complex hypotheses in later sections, the partial F test will be illustrated in the simple case of two quantitative independent variables.

As stated above, each regression coefficient, b_k, in multiple regression represents the partialed effect of that variable, X_k, upon Y while controlling for the other Xs. For example, the achievement data regression model illustrates this:

$$Y = b_0 + b_1X_1 + b_2X_2 + e. \quad (5)$$

Thus, b_1 represents the partial effect of X_1 controlling for X_2. Another measure of the partial effect of a given X upon Y is $\Delta R^2_{(X_k)}$, where X_k is the variable for which the partial effect is desired. The value for $\Delta R^2_{(X_k)}$ is derived as follows:

$$\Delta R^2_{(X_k)} = R^2_f - R^2_r$$

where the subscripts for R^2_f and R^2_r refer to full (f) and reduced (r) respectively; R^2_f is the R^2 value from the full regression model (the model that includes all Xs, such as [5] above); and R^2_r is the R^2 value of the reduced model. A reduced model contains only a select number of Xs, such as the model below which omits X_1:

$$Y = b_0 + b_2X_2 + e. \quad (6)$$

Both models, the full and reduced, will likely have different values for R^2, and the full regression model R^2, R^2_f, will always be greater than (or equal to) R^2_r, which is the R^2 value for the reduced regression model. Since $\Delta R^2_{(X_k)}$ is the difference between two R^2 values, $\Delta R^2_{(X_k)}$ represents the increment or increase in R^2 due to adding a given X_k (or a set of Xs). Therefore, $\Delta R^2_{(X_k)}$ represents the partial increase (or partial effect) in the model R^2 attributable to an X_k (or a set of Xs). This can be seen in Table 2 below. As Table 2 illustrates, the increment in the overall model R^2 due to adding X_1 is .10; that is, a .10 increase is observed in R^2_f once X_1 is added to the regression equation.

Table 2: *Calculating $\Delta R^2_{(X_k)}$*

Model	Equation	R^2 Values
Full Model	$Y = b_0 + b_1X_1 + b_2X_2 + e.$	$R^2_f = .40$
Reduced Model (X_1 omitted)	$Y = b_0 + b_2X_2 + e.$	$R^2_r = .30$
		$\Delta R^2_{(X_1)} = .4 - .3 = .10$

As a further illustration, suppose one wishes to calculate the partial effect of time spent studying, X_1, on achievement, Y, using $\Delta R^2_{(X_1)}$. The value of $\Delta R^2_{(X_1)}$ will represent the increase in the overall model R^2 that is attributable to X_1 over and above the contribution of X_2. The reason that $\Delta R^2_{(X_1)}$ represents the increase over X_2 is because X_2 is included in the model from the outset. The model with only X_2 is the reduced regression model. The model with both X_1 and X_2 is the full regression model.

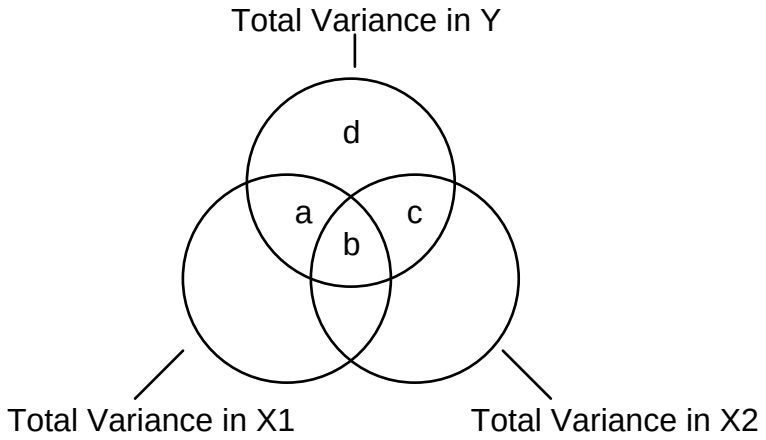
The full model is represented by (5). The R^2 for this model, using data in Table 1, is .8742. Since (5) is the full model, $R^2_f = .8742$. The R^2 for the reduced model, equation (6), is .7501, and since this is the reduced model, $R^2_r = .7501$. The resultant $\Delta R^2_{(X_1)}$ is

$\Delta R^2_{(X_1)} = R^2_f - R^2_r = .8742 - .7501 = .1241.$

So the increment in R^2 attributable to time spent studying is .1241, and this value represents the increase in the overall model R^2 that is unique to time spent studying.

The value of $\Delta R^2_{(X_k)}$ is also referred to as the *semi-partial* or *part* correlation, or, more correctly, the squared value of the semi-partial correlation. The semi-partial correlation can best be illustrated using a Venn diagram, such as the one presented in Figure 1.

Figure 1



The total variance in Y is composed of four parts: a, b, c, and d. The total area in Y covered by X_1 and X_2—a, b, and c—represents the total variance explained by X_1 and X_2, i.e., this area, in terms of proportions, is represented by the model R^2. Note that areas a and c are the unique amounts of variance in Y explained by X_1 and X_2, respectively. The area denoted as b is the variance in Y that is jointly explained by both X_1 and X_2, and the area represented by d is the variance that is not explained, i.e., 1 -

R^2. Since a is the proportion of variance in Y explained solely by X_1, this proportion is represented by $\Delta R^2_{(X1)}$. Similarly, the proportion of area explained by X_2, denoted c, is equal to $\Delta R^2_{(X2)}$.

The calculation of semi-partial correlations may be done either through the method of finding explained above, or through Pearson correlations, r. For more information on semi-partial correlations in terms of Pearson's r, see Cohen and Cohen (1983, pp. 88-92) or an introductory statistics text, such as Glass and Hopkins (1984).[1]

Once partial effects, in terms of $\Delta R^2_{(Xk)}$, are obtained, one must next test the null hypothesis that the partial effect equals zero, i.e.,

H_0: $\Delta R^2_{(Xk)} = 0.00$.

If H_0 is not rejected, then one may conclude that the variable X_k does not statistically affect Y; that X_k does not statistically add to the model explained variation in Y. When X_k is one independent variable or one vector, the null hypothesis for the partial effect expressed as $\Delta R^2_{(Xk)}$ is identical to the null hypothesis for the corresponding regression coefficient, b_k, i.e.,

H_0: $\Delta R^2_{(Xk)} = 0.00$ and

H_0:$\beta_k = 0.00$.

In other words, the two nulls express the same thing. From earlier discussion of regression, it was shown that to test H_0:$\beta_k = 0.00$ one may use the t-test. To test H_0: $\Delta R^2_{(Xk)} = 0.00$ generally requires the use of the partial F test. The F statistic for the partial F test is calculated as

$$F = \frac{\Delta R^2_{(Xk)}/(df_{2r} - df_{2f})}{(1 - R^2_f)/df_{2f}} \qquad (7)$$

where

$\Delta R^2_{(Xk)}$ is the partial effect to be tested;
df_{2f} is the error degrees of freedom for the *full* model ($n - k_f - 1$);
df_{2r} is the error degrees of freedom for the *reduced* model ($n - k_r - 1$); and
R^2_f is the *full* model R^2 value.

The obtained F value from (7) is tested against a critical F value with degrees of freedom equal to

$df_1 = df_{2r} - df_{2f}$,

and

$df_2 = df_{2f}$.

Referring again to the achievement data, the test of the partial effect for X_1 (study time) is illustrated below. Recall that the full model is

---

[1] Cohen, J. & Cohen, P. (1983). Applied multiple regression/correlation analysis for the behavioral sciences (2nd ed.). Hillsdale, NJ: Erlbaum.
Glass, G., & Hopkins, K. (1984). Statistical methods in education and psychology (2nd ed.). Boston, MA: Allyn and Bacon.

$Y = b_0 + b_1X_1 + b_2X_2 + e,$

and the reduced model is

$Y = b_0 + b_2X_2 + e.$

The error degrees of freedom for the full model is

$df_{2f} = n - k_f - 1 = 12 - 2 - 1 = 9.$

The error degrees of freedom for the reduced model is

$df_{2r} = n - k_r - 1 = 12 - 1 - 1 = 10.$

The R^2 value for the full model, reported above, is $R^2_f = .8742$. Using this information, the partial F ratio may now be obtained as follows:

$$F = \frac{\Delta R^2_{(X_k)}/(df_{2r} - df_{2f})}{(1 - R^2_f)/df_{2f}}$$

$$= \frac{.1241/(10 - 9)}{(1 - .8742)/9} = \frac{.1241}{(.1258)/9} = \frac{.1241}{0.01398} = 8.877.$$

This F value is compared against a critical F value with the following degrees of freedom:

$df_1 = df_{2r} - df_{2f} = 10 - 9 = 1$

and

$df_2 = df_{2f} = 9.$

The critical F value is ${}_{.05}F_{1,9} = 5.12$. Since the calculated F ratio is greater than the critical F ratio, $8.877 > 5.12$, the null hypothesis, $H_0: \Delta R^2_{(X_k)} = 0.00$, is rejected and one may conclude that time spent studying, X_1, does significantly contribute to explained variation in achievement.

The partial F test will be most useful when multiple regression with categorical variables are examined.

Inferential Procedures for Regression Coefficients

If the overall null hypothesis, $H_0: R^2 = 0.00$, is rejected, the next step in multiple regression (and ANOVA) is to examine the individual variables for statistical significance. As noted above, in multiple regression one is interested in testing whether there is a partial relationship between the k^{th} X and Y, controlling for the other Xs in the model. The null hypothesis states that their is no partial relationship between X_k and Y, that is,

$H_0: \beta_k = 0.00.$

If the null is rejected, then one may conclude that a partial relationship does exist between X_k and Y. A non-directional alternative hypothesis states that a partial relationship does exist, thus:

H_1: $\beta_k \neq 0.00$.

As with simple regression, each regression coefficient has a corresponding standard error (SE_{bk}). The ratio of the partial regression coefficient to its SE provides a t-ratio. As with the two-group t-test and simple regression, the calculated t-ratio may be compared against a critical t ratio to determine statistical significance. Or, one may simply use p-values obtained for each coefficient to determine statistical significance.

For example, in the achievement data the estimated partial effect of time spent studying on achievement is $b_1 = 1.30$, and the SE for b_1 is $SE_{b1} = .437$, so the t ratio for b_1 is:

$t = b_1 / SE_{b1} = 1.30 / .437 = 2.975.$

The critical t value, using an α of .05, and df = n - k - 1, is $_{.05}t_9 = \pm 2.262$, so the null hypothesis is rejected:

If $|t| \geq t_{crit}$ reject H_0, otherwise fail to reject H_0.

If 2.98 ≥ 2.262 *reject* H_0, otherwise fail to reject H_0.

In terms of p-values, the p-value for the obtained t ratio of 2.98 is $p = .015$. The usual decision rule, for non-directional tests, applies:

If $p \leq \alpha$, reject H_0, otherwise fail to reject H_0,

so, since the obtained *p* is less than alpha, H_0 is rejected:

If .015 ≤ .05, reject H_0, otherwise fail to reject H_0,

and one may conclude that a positive relationship exists between time spent studying and achievement, holding constant the effects of ability. Both b_0 and b_2 may be tested in a similar fashion.

Interval Estimation: Confidence Intervals (CI)

Recall that the CI represents the upper and lower bound to the point estimate of the regression coefficients. Thus, the CI represents, with a set level of precision, a range of possible values for b_k, and therefore the CI provides some indication of the exactness or precision with which sample estimates are derived from the sample data.

As with simple regression, the CI for b_1 may be formed as:

$b_1 \pm t_{(\alpha/2,df)}SE_{b1}$

where t is the critical t value obtained from a table of t values representing a two-tailed alpha (α) level (such as .05) with degrees of freedom equal to n-k-1, and SE_{b1} is the standard error of b_1 described above.

For the current example, the 95% confidence interval (.95CI) for b_1 is

.95CI: $b_1 \pm t_{(\alpha/2,df)}SE_{b1}$

.95CI: $1.302 \pm (2.262)(0.437)$

.95CI: 1.302 ± 0.988

.95CI: (2.29, 0.314).

CIs for b_0 and b_2 are constructed in a similar fashion.

Such a CI enables the researcher to state that one may be 95% confident that the true population coefficient may be as high as 2.29 or as low as 0.314. As this illustrates, CIs provide a sense of precision that point estimates do not.

Factors Affecting Power and Precision

As with simple regression, the notable factors in multiple regression that directly influence both power and precision are (a) sample size, n, and (b) residual variation, $MSE = \sum(Y - Y')/(n-k-1)$, and/or *SEE* (standard error of estimate: $SEE = \sqrt{MSE}$). Sample size represents the one factor that researchers typically have the most control over in research. As sample size increases, both power and precision also increase. Remember that increases in precision results in *decreases* in CIs. Note that better fitting models of Y also have smaller residual variation, thus as *SEE* or *MSE* are reduced, both power and precision increase. One difference between simple regression and multiple regression is that with the latter better fitting models can be obtained, therefore it is more likely that *MSE* can be directly affected by the researcher.

Standardized Regression Equation

The population and sample regression equations, respectively, are

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_i,$$

and

$$Y_i = b_0 + b_1 X_1 + b_2 X_2 + e_i.$$

The coefficients for these equations are referred to as unstandardized partial regression coefficients. The exact value of, for instance, b_1 will change when the scale of X is changed. This may be problematic because most variables in social and educational research take many different scales, thus it becomes difficult to assess whether a given coefficient represents a large or small effect. That is, for a given X_k, which coefficient represents the larger impact on Y, .01 or 100.00? Without knowledge of the scale of measurement, it is difficult to tell.

The standardized regression equation enables one to compare the effects of two quantitative variables in an attempt to determine which variable has the more substantial impact upon Y. The standardized regression equation is:

$$Z'_y = \beta_1 Z_{X1} + \beta_2 Z_{X2}$$

or

$$Z'_y = P_1 Z_{X1} + P_1 Z_{X1}$$

where

Z'_y is the predicted value of Y in Z scores; β_1 and P_1 represent the standardized partial regression coefficient for X_1; β_2 and P_2 represent the standardized partial regression coefficient for X_2; and Z_{X1} and Z_{X2} are the Z score values for the variables X_1 and X_2, respectively.

Once the regression equation is standardized, then the partial effect of a given X upon Y, or Z_x upon Z_y, becomes somewhat easier to interpret. For the current example, the standardized solution is:

$$Z'_y = P_1 Z_{X1} + P_1 Z_{X1}$$

$$= 0.400(Z_{X1}) + 0.677(Z_{X1})$$

The standardized partial coefficient represents the amount of change in Z_y for a *standard deviation* change in Z_x. So, if X_1, time spent studying, were increased by one standard deviation, then one would anticipate a 0.40 standard deviation increase in achievement, holding constant the effect of ability.

Finally, note that a standardized regression equation is NOT appropriate when categorical (qualitative) IVs are present.

Obtaining and Reporting Multiple Regression Results

Regression results can be obtained from SPSS using the same command as used for simple regression. One simply adds additional Xs into the independent variables command section. Sample output from the achievement data is provided below.

```
Multiple R            .93499
R Square              .87421
Adjusted R Square     .84626
Standard Error       3.80703

Analysis of Variance
                    DF        Sum of Squares      Mean Square
Regression           2             906.55875        453.27937
Residual             9             130.44125         14.49347

F =      31.27472       Signif F =  .0001

---------------------- Variables in the Equation -----------------------

Variable             B         SE B     95% Confdnce Intrvl B       Beta
TIME          1.302289     .437043      .313629     2.290948     .399660
ABILITY       2.524210     .499843     1.393487     3.654934     .677329
(Constant)   63.901747    2.835634    57.487101    70.316394

----------- in ------------

Variable           T  Sig T
TIME           2.980  .0155
ABILITY        5.050  .0007
(Constant)    22.535  .0000
```

The independent variables are time spent studying, TIME, and academic ability, ABILITY. The outcome variable is achievement, ACH. Note all components previously discussed, such as R, R^2, and adj. R^2; the regression coefficients b_0 (denoted as constant), b_1 (B for TIME), and b_2 (B for ABILITY); the ANOVA summary table with SSR, SSE, and MSE; and the inferential tests—overall F test (F = 31.27472, p = .0001), and t tests for the partial coefficients.

Reporting results may take several forms. Most common is a tabular display, although for models with few IVs, reporting results within the text of your manuscript may be feasible. The tabular format will be illustrated.

Table 1. Descriptive Statistics and Correlations among Achievement, Time, and Ability

Variable	Correlations		
	Achievement	Time	Ability
Achievement	---		
Time	.720*	---	
Ability	.866*	.472	---
Mean	84.500	4.833	5.667
SD	9.709	2.980	2.605

Note. n = 12
* p < .05

Table 2. Regression of Achievement on Time Spent Studying and Academic Ability

Variable	b	se	β	ΔR^2	95%CI	t
Time	1.30	0.437	0.400	.124	0.31, 2.29	2.98*
Ability	2.52	0.500	0.677	.356	1.39, 3.65	5.05*
Intercept	63.90	2.836	na	na	57.49, 70.32	22.54*

Note. R^2 = .874, adj. R^2 = .846, F = 31.27*, df = 1,9, MSE = 14.49, n = 12. The symbol ΔR^2 represents the semi-partial correlation squared.
*p < .05.

Correlations show that achievement is positively, and strongly, related to both time spent studying and academic ability. Regression results also show that both predictors, considered simultaneously, are statistically related to achievement. Based upon the squared-part correlations (ΔR^2) and the standardized regression coefficients, academic ability appears to be the stronger of the two predictors of achievement. In summary, the more time spent studying and the higher one's academic ability, the greater one's achievement.

Exercises

(1) A researcher wishes to know whether number of hours studied at home is related to general achievement amongst high school students. The student ability needs to be controlled to better assess the effects of studying.

Student	High School GPA	IQ	Time Spent Studying Per Week (in Hours)
Bill	3.33	117	3
Bob	1.79	90	5
Stewart	2.21	101	12
Linda	3.54	121	9
Lisa	2.89	105	11
Ann	2.54	110	1
Fred	2.66	112	0
Carter	1.10	85	3
Kathy	3.67	128	2

(2) Does SAT adequately predict college success, once rank is controlled?

Student	Freshmen Collegiate GPA	HS Rank*	SAT Scores
Bill	3.33	52	1000
Bob	1.79	233	750
Stewart	2.21	150	890
Linda	3.54	43	1100
Lisa	2.89	95	900
Ann	2.54	43	860
Fred	2.66	120	1010
Carter	1.10	280	640
Kathy	3.67	33	1240

*Out of 300 students.

(3) A teacher is convinced that frequency of testing within her classroom increases student achievement. She runs an experiment for several years in her algebra class. The frequency in which she presents tests to the class varies across quarters. For example, one quarter students are tested only once during the term, while in another quarter students are tested once every week. Is there evidence that testing frequency is related to average achievement?

Quarter	Testing Frequency During Quarter	Average IQ in Class	Overall Class Ach. on Final Exam
Fall 1991	1	105	85.5
Winter 1992	2	108	86.5
Spring 1992	3	108	88.9
Summer 1992	4	109	89.1
Fall 1992	5	107	87.2
Winter 1993	6	110	90.5
Spring 1993	7	108	89.8
Summer 1993	8	114	92.5
Fall 1994	9	110	89.3
Winter 1994	10	112	90.1

(4) An administrator wishes to know whether a relationship exists between the number of tardies or absences a student records during the year and that student's end-of-year achievement as measured by GPA. The administrator randomly selects 12 students and collects the appropriate data. The principal also has standardized ITBS test scores for each student.

Student	GPA	ITBS	Tardies/Absences
Bill	3.33	65	2
Bob	1.79	40	10
Stewart	2.21	50	5
Linda	3.54	70	6
Lisa	2.89	49	3
Ann	2.54	55	4
Fred	2.66	58	6
Carter	1.10	37	12
Bill	3.10	55	3
Sue	2.10	45	8
Loser	2.31	51	6
Kathy	3.67	63	2

Results for Exercises

Statistical results are presented below so you may check for accuracy of findings. Note that results formats used below do not follow the format specified above and in the notes on Reporting Statistical Outcomes.

(1)

Table 1 *Descriptive Statistics*

Variable	Correlations		
	GPA	Time	IQ
GPA	1.000		
Time	.033	1.000	
IQ	.963	-.149	1.000
Mean	2.637	5.11	107.67
SD	.844	4.46	14.053

n = 9

Table 2 *Regression of GPA on Time Spent Studying and IQ*

Variable	B	SE B	β	ΔR^2	.95CI
Time	.034	.015	.18	.031	-.005, .07
IQ	.059	.005	.989	.957*	.047, .071
Intercept	-3.93	.56			

Note. R^2 = .958, adj. R^2 = .945, p = .0001, n = 12, $F_{2,6}$ = 70.09, *MSE* = 0.038. The term ΔR^2 represents the semi-partial correlation squared.

*p < .05.

(2)

Table 1 *Descriptive Statistics*

Variable	Correlations		
	GPA	Rank	SAT
GPA	1.000		
Rank	-.928	1.000	
SAT	.935	-.827	1.000
Mean	2.637	116.556	932.22
SD	.844	89.33	180.331

n = 9

Table 2 *Regression of GPA on Rank and SAT*

Variable	B	SE B	β	ΔR^2	.95CI
Rank	-.004	.001	-.49	.075*	-.008, -.00009
SAT	.002	.00007	.53	.088*	.0006, .004
Intercept	.864	.861			

Note. R^2 = .950, adj. R^2 = .933, p = .0001, n = 12, $F_{2,6}$ = 57.42, *MSE* = 0.047. The term ΔR^2 represents the semi-partial correlation squared.

*p < .05.

(3)

Table 1 *Descriptive Statistics*

Variable	Correlations		
	Final	Testing	IQ
Final	1.000		
Testing	.752	1.000	
IQ	.887	.767	1.000
Mean	88.94	5.50	109.10
SD	2.059	3.03	2.255

n = 10

Table 2 *Regression of Achievement on Testing Frequency and IQ*

Variable	B	SE B	β	ΔR^2	.95CI
Testing	.117	.179	.17	.012	-.306, .542
IQ	.607	.212	.75	.233*	.104, 1.11
Intercept	22.06	22.44			

Note. R^2 = .80, adj. R^2 = .74, p = .0036, n = 12, $F_{2,7}$ = 13.94, *MSE* = 1.09. The term ΔR^2 represents the semi-partial correlation squared.

*p < .05.

(4)

Table 1 *Descriptive Statistics*

Variable	Correlations		
	GPA	ITBS	Tardies
GPA	1.000		
ITBS	.917	1.000	
Tardies	-.851	-.717	1.000
Mean	2.60	53.167	5.583
SD	.756	9.925	3.147

n = 12

Table 2 *Regression of GPA on Tardies and ITBS*

Variable	B	SE B	β	ΔR^2	.95CI
ITBS	.048	.01	.63	.194*	.024, .071
Tardies	-.095	.032	-.39	.077*	-.17, -.02
Intercept	.580	.700			

Note. R^2 = .918, adj. R^2 = .899, p = .0036, n = 12, $F_{2,9}$ = 50.47, *MSE* = 0.057. The term ΔR^2 represents the semi-partial correlation squared.

*p < .05.