Homework for z scores, percentile ranks, and z-tests

There are six activities below. Answers to each can be found following the activities.

1. Below are scores from test 1 of a section of EDUR 7130. What is the percentile rank for each of these scores: 72.5, 82.5, 85?

Test 1 Grades in EDUR 7130

82.5	72.5	90	97.5	95	85	95	97.5	92.5
95	60	78.75	87.5	92.5	77.5	85	78.75	95
95	82.5	85	100	85	73.75	72.5	92.5	76.25
97.5	80	97.5	91.25	93.75	85	66.25		

2. Male and Female students from a graduate class were asked their opinion about a particular textbook. The scale on which they replied ranged from 1=strongly unfavorable to 5=strongly favorable. For each group, convert these scores to z scores.

Male Opinion Scores		Female Opinion Scores	
1	5	5	1
5	4	3	2
4	3	1	1

3. Convert z-scores to raw scores (X = M + Z*SD):

(a) IQ scores with M = 100, SD = 15, what are the raw scores for these z scores?
z = 1.5
z = .93
z = -3.2

(b) EDUR 8131 test 1 scores with M = 83.5, SD = 8.4, what are the raw scores for these z scores?
z = 2.1
z = -1.9
z = -2.7

4. Below are z-scores. Determine the proportion of scores that:

(a) fall between the mean (z = 0.00) and 1.33; between 0.00 and -.93?
(b) are less than or equal to 1.5; less than or equal to -.75?
(c) are greater than or equal to 1.4; greater than or equal to -.83?
(d) are between -1.0 and 1.5; between 1.0 and 1.3; between -1.3 and -1.1?
(e) are greater than 1.96 or less than -1.96;
(f) < -2.576 or > 2.576 (may need to round to 2.58 if your table does not show 2.576)?

5. Assuming that SAT scores are normally distributed with a M = 500 and SD = 100, provide answers to the following:

(a) find the proportion of scores between 427 and the mean;

(b) find the proportion of scores below 627;

(c) find the proportion of scores above 427;

(d) find the proportion of scores between 420 and 380;

(e) find the proportion of scores between 540 and 380.

6. Determine the appropriate percentile ranks for each:

(1) What is the PR for z = -0.33?

(2) What is the PR for z = 1.20?

(3) What is the PR for z = 0.00?

(4) What is the PR for z = -2.00?

7. Male and Female students from a graduate class were asked their opinion about a particular textbook. The scale on which they replied ranged from 1=strongly unfavorable to 5=strongly favorable.

   Graduate students across the country are believed to hold opinions regarding this textbook with the following parameters:

   Males $\mu = 2.5$ and $\sigma = .5$, while females $\mu = 2.2$ and $\sigma = .75$

   Thus females tend to hold a less favorable opinion toward this textbook than males (population means 2.5 vs. 2.2).

   Is there any evidence that the sample of males from the graduate class, reported below, hold an opinion that differs from other males across the country? Is there any evidence that sampled females hold an opinion that differs from females across the county? What are the written and symbolic null and nondirectional hypotheses for these two questions?

Male Opinion Scores		Female Opinion Scores	
1	5	5	1
5	4	3	2
4	3	1	1

Answers

1. What is the percentile rank for each of these scores: 72.5, 82.5, 85?

Percentile ranks can be determined by using the cumulative percent column provided by SPSS.

PR of 72.5 = 11.8
PR of 82.5 = 35.3
PR of 85.0 = 50.0

Test Scores

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	60.00	1	2.9	2.9	2.9
	66.25	1	2.9	2.9	5.9
	72.50	2	5.9	5.9	11.8
	73.75	1	2.9	2.9	14.7
	76.25	1	2.9	2.9	17.6
	77.50	1	2.9	2.9	20.6
	78.75	2	5.9	5.9	26.5
	80.00	1	2.9	2.9	29.4
	82.50	2	5.9	5.9	35.3
	85.00	5	14.7	14.7	50.0
	87.50	1	2.9	2.9	52.9
	90.00	1	2.9	2.9	55.9
	91.25	1	2.9	2.9	58.8
	92.50	3	8.8	8.8	67.6
	93.75	1	2.9	2.9	70.6
	95.00	5	14.7	14.7	85.3
	97.50	4	11.8	11.8	97.1
	100.00	1	2.9	2.9	100.0
	Total	34	100.0	100.0	

2. Male and Female students from a graduate class were asked their opinion about a particular textbook. The scale on which they replied ranged from 1=strongly unfavorable to 5=strongly favorable. For each group, convert these scores to z scores.

Using the deviation method for calculating SD and VAR for male opinion scores, z scores can be calculated (see table below):

$$z = \frac{(X - M)}{SD}$$

For males, z scores would be:

X	M	X-M	$(X-M)^2$	z
1.000	3.667	-2.667	7.113	-2.667 / 1.506 = -1.77
5.000	3.667	1.333	1.777	1.333 / 1.506 = 0.885
4.000	3.667	0.333	0.111	0.333 / 1.506 = 0.221
5.000	3.667	1.333	1.777	1.333 / 1.506 = 0.885
4.000	3.667	0.333	0.111	0.333 / 1.506 = 0.221
3.000	3.667	-0.667	0.445	-0.667 / 1.506 = -0.443

SS (sum of squares) = $\Sigma(X-M)^2$ = (7.113 + 1.777 + 0.111 + 1.777 + 0.111 + 0.445) = 11.334

VAR = $s^2 = \Sigma(X-M)^2/(n-1)$ = SS/(n-1) = 11.334/(6-1) = 11.334/5 = 2.2668

SD = s = $\sqrt{var}$ = $\sqrt{2.2668}$ = 1.506

For females, z scores would be:

X	M	X-M	$(X-M)^2$	z
5.000	2.167	2.833	8.026	2.833 / 1.602 = 1.768
3.000	2.167	0.833	0.694	0.833 / 1.602 = 0.52
1.000	2.167	-1.167	1.362	-1.167 / 1.602 = -0.728
1.000	2.167	-1.167	1.362	-1.167 / 1.602 = -0.728
2.000	2.167	-0.167	0.028	-0.167 / 1.602 = -0.104
1.000	2.167	-1.167	1.362	-1.167 / 1.602 = -0.728

SS (sum of squares) = $\Sigma(X-M)^2$ = (8.026 + 0.694 + 1.362 + 1.362 + 0.028 + 1.362) = 12.834

VAR = $s^2 = \Sigma(X-M)^2/(n-1)$ = SS/(n-1) = 12.834/(6-1) = 12.834/5 = 2.5668

SD = s = $\sqrt{var}$ = $\sqrt{2.5668}$ = 1.602

3. Convert z-scores to raw scores:

To convert from z to raw scores, use this formula X = $M + (z * SD)$

(a) IQ scores with M = 100, SD = 15, what are the raw scores for these z scores?

z = 1.5; X = 100 + (1.5 * 15) = 122.5

z = .93; X = 100 + (.93 * 15) = 113.95

z = -3.2; X = 100 + (-3.2 * 15) = 52

(b) EDUR 8131 test 1 scores with M = 83.5, SD = 8.4, what is the raw scores for these z scores?

z = 2.1; X = 83.5 + (2.1 * 8.4) = 101.14

z = -1.9; X = 83.5 + (-1.9 * 8.4) = 67.54

z = -2.7; X = 83.5 + (-2.7 * 8.4) = 60.82

4. Below are z-scores. Determine the proportion of scores that:

(a) fall between the mean (z = 0.00) and 1.33; between 0.00 and -.93?

0.00 to 1.33 = .4082

0.00 to -.93 = .3238

(b) are less than or equal to 1.5; less than or equal to -.75?

$x \leq 1.5$ = .9332

$x \leq -.75$ = .2266

(c) are greater than or equal to 1.4; greater than or equal to -.83?

$x \geq 1.4$ = .0808

$x \geq -.83$ = .7967

(d) are between -1.0 and 1.5; between 1.0 and 1.3; between -1.3 and -1.1?

-1.0 to 1.5 = .7745

-1.3 to -1.1 = .0389

(e) are greater than 1.96 or less than -1.96

$x \leq -1.96$ or $1.96 \geq x$ = .05

(f) < -2.576 or > 2.576?

$x \leq -2.576$ or $2.576 \geq x$ = .01

5. Assuming that SAT scores are normally distributed with a M = 500 and SD = 100, provide answers to the following:

To do each of these, one must first convert raw scores to z scores, then find the appropriate area under the normal curve as done in exercise 4 above.

(a) find the proportion of scores between 427 and the mean;

z score for 427 = (427-500)/100 = -73/100 = -.73

z score for mean = (500-500)/100 = 0.00

Proportion of scores between -.73 and 0.00 = .2673

(b) find the proportion of scores below 627;

z score for 627 = 1.27

Proportion of scores below 1.27 = .8980

(c) find the proportion of scores above 427;

z score for 427 = -.73

Proportion of scores above -.73 = .7673

(d) find the proportion of scores between 420 and 380;

z score for 420 = -.80

z score for 380 = -1.20

Proportion of scores between -1.20 and -.80 = .0968

(e) find the proportion of scores between 540 and 380.

z score for 540 = .40
z score for 380 = -1.20
Proportion of scores between -1.20 and .40 = .5403

6. Determine the appropriate percentile ranks for each:

To answer, find the proportion of scores that are below the given z score, multiple that proportion by 100 and the product provides the PR.

(1) What is the PR for z = -0.33?

Proportion of scores that lie below -.33 = .3707, so PR = .3707*100 = 37.07

PR = 37.07

(2) What is the PR for z = 1.20?

PR = 88.49

(3) What is the PR for z = 0.00?

PR = 50

(4) What is the PR for z = -2.00?

PR = 2.28

7. Graduate students across the country are believed to hold opinions regarding this textbook with the following parameters:

   Males $\mu = 2.5$ and $\sigma = .5$, while females $\mu = 2.2$ and $\sigma = .75$

   Thus males tend to hold a slightly more favorable opinion toward this textbook than females (population means 2.5 vs. 2.2).

   Is there any evidence that the sample of males from the graduate class, reported below, hold an opinion that differs from other males across the country? Is there any evidence that sampled females hold an opinion that differs from females across the county? What are the written and symbolic null and nondirectional hypotheses for these two questions?

Male Opinion Scores		Female Opinion Scores	
1	5	5	1
5	4	3	2
4	3	1	1

<u>Males</u>

Null:

H_0: $\mu = 2.5$

There is no difference in textbook opinions between males in this sample and males across the county.

Nondirectional:

H_1: $\mu \neq 2.5$

There is a difference in textbook opinions between males in this sample and males across the county.

z-test:

Calculate z and find appropriate proportion of cases associated with z calculated. Since a nondirectional alternative hypothesis is specified, the proportion found on one side of the distribution must be multiplied by two to cover both sides of the normal distribution since the test simply asks if the sample differs (which could theoretically differ by being higher or lower than the population mean).

The mean for males in this sample is M = 3.667

$$z = \frac{(M-\mu)}{(\sigma/\sqrt{n})} = \frac{(3.667-2.5)}{(.5/\sqrt{6})} = \frac{1.167}{(.5/2.449)} = \frac{1.167}{.204} = 5.72$$

What proportion of z scores will be greater than 5.72? It will be .0001 or less. Similarly, what proportion of z scores will be less than -5.72? Again, .0001 or less. Combined these two proportions provide the p-value for the z-test of .0001+.0001 = .0002. If alpha (α) = .05, then we reject the null hypothesis since the p-value for this z-test is less than α, that is, .0002 < .05, so reject H_0.

We conclude that this sample of males hold opinions about the textbook that differs from the population of males. Specifically, this sample of males holds an opinion that is statistically more favorable than males across the country.

<u>Females</u>

Null:

H_0: $\mu = 2.2$

There is no difference in textbook opinions between females in this sample and females across the county.

Nondirectional:

H_1: $\mu \neq 2.2$

There is a difference in textbook opinions between females in this sample and females across the county.

z-test:

The mean for females in this sample is M = 2.167

$$z = \frac{(M-\mu)}{(\sigma/\sqrt{n})} = \frac{(2.167-2.2)}{(.75/\sqrt{6})} = \frac{-.033}{(.75/2.449)} = \frac{-.033}{.306} = -0.11$$

What proportion of z scores will be less than -.11? It will be .4562. Similarly, what proportion of z scores will be greater than .11? Again, .4562 since the normal distribution is symmetrical. Combined these two proportions provide the p-value for the z-test of .4562+.4562 = .9124. If alpha (α) = .05, then we fail to

reject the null hypothesis since the p-value for this z-test is greater than α, that is, .9124 > .05, so fail to reject H_0.

We conclude that this sample of females hold opinions about the textbook that does not seem to differ from the population of females. Specifically, this sample of females holds an opinion that generally unfavorable toward this textbook, and that is an opinion held by most females across the country.