Central Limit Theorem Illustrated

Central Limit Theorem: If one is sampling independently from a population that has mean of μ and variance of σ^2, then as the sample size n approaches infinity, the sampling distribution of the sample mean $\overline{X}$ approaches normality without regard to the shape of the sampled population.

In practice, this means that the larger the sample size used, the better the sampling distribution of M will approximate a normal distribution. Note, however, that the Central Limit Theorem applies only the $\overline{X}$. Below are several histograms that display a population distribution of SAT scores, and several sampling distributions for means, $\overline{X}$, taken from 5,000 random samples drawn from the population distribution. Figure 1 shows a histogram of 5,00 simulated SAT scores. Note that the distribution of SAT scores is bimodal. An outline of the normal curve is drawn on this figure as a reference to illustrate that the distribution of SAT scores is far from normal.

Figures 2 through 9 show histogram, not of raw SAT scores, but of means from samples of differing sizes. Figure 2, for example, shows means taken from a sample size of 2. To construct Figure 2, a total of 5,000 samples (n = 2 for each sample) of SAT scores were taken from those SAT scores displayed in Figure 1. For Figure 3, another set of 5,000 samples was taken from SAT scores, but with a sample size of 3. Each successive figure shows distribution of sample means for varying sample sizes. Thus, Figures 2 through 9 are histograms of sampling distributions for the mean. Note that as sample sizes increase, the shape of the sampling distribution of means approaches a normal curve and looks less and less like the bimodal distribution of raw SAT scores. This is exactly what the central limit theorem predicts.