

## Notes 8b Multiple Regression

### 1. Regression Equation

New components due to use of *multiple* regression, rather than *simple* regression, in green

$$Y = b_0 + b_1 X_1 + b_2 X_2 + e$$

$$Y' = b_0 + b_1 X_1 + b_2 X_2 \text{ (prediction equation, used to obtain predicted value of } Y)$$

$b_0$  = predicted value of  $Y$  when both  $X_1$  and  $X_2$  take the value of 0.00

$b_1$  = indicates how much mean change in  $Y$  is expected for a 1 unit change in  $X_1$ , controlling for  $X_2$

$b_2$  = indicates how much mean change in  $Y$  is expected for a 1 unit change in  $X_2$ , controlling for  $X_1$

$e$  = error or residual term – deviation between  $Y$  and  $Y'$ , i.e.,  $Y - Y'$

$Y'$  = predicted  $Y$  using the regression equation

### 2. Literal Interpretation of Coefficients and Predicted Values

$b_0$  = predicted value of  $Y$  when  $X_1 = 0.00$  and  $X_2 = 0.00$

$b_1$  = for each 1 unit increase in  $X_1$ , the mean of  $Y$  is expected to change by  $b_1$  controlling for  $X_2$

$b_2$  = for each 1 unit increase in  $X_2$ , the mean of  $Y$  is expected to change by  $b_2$  controlling for  $X_1$

### Examples

(a)  $X$  and  $Y$  Interpretation

$$b_0 = 9.00$$

$$b_1 = -0.33$$

$$b_2 = 1.45$$

Prediction Equation

$$Y' = b_0 + b_1 X_1 + b_2 X_2$$

$$Y' = 9.00 + -0.33 (X_1) + 1.45 (X_2)$$

Interpretation of Coefficients

$b_0 = 9.00$ :  $Y$  is expected to be 9.00 when both  $X_1$  and  $X_2$  are 0.00

$b_1 = -0.33$ : for each 1 point increase in  $X_1$ ,  $Y$  is expected to change by  $-0.33$  controlling for  $X_2$

$b_2 = 1.45$ : for each 1 point increase in  $X_2$ ,  $Y$  is expected to change by 1.45 controlling for  $X_1$

(b) Math Achievement and Test Anxiety and Hours Studied

IV 1 = Test Anxiety

IV 2 = Hours Studied

DV = Math Achievement

Prediction Equation

Math Achievement' =  $b_0 + b_1$  (Test Anxiety) +  $b_2$  (Hours Studied)

Math Achievement' =  $9.00 + -0.33$  (Test Anxiety) +  $1.45$  (Hours Studied)

*Literal Interpretation of Coefficients*

$b_0 = 9.00$ :

when both test anxiety and hours studied are 0.00, math achievement is expected to be 9.00

$b_1 = -0.33$  (test anxiety):

for each 1 point increase in test anxiety, math achievement changes by  $-0.33$  controlling for hours studied

$b_2 = 1.45$  (hours studied):

for each 1 hour increase in study time (hours studied), math achievement is expected to change (increase) by 1.45 controlling for test anxiety

Another example for interpretation of coefficients

DV = money earned

IV = hours worked per week

Money earned' =  $b_0 + b_1$ (hours worked)

Coefficient values

$b_0 = \$0.00$

$b_1 = \$10$

Money earned' =  $b_0 + b_1$ (hours worked)

Money earned' =  $\$0.00 + \$10$ (hours worked)

Literal Interpretation

$b_0$  = if you don't work any hours, you money earned would be  $\$0.00$  (no work, no pay)

$b_1$  = for each additional hour worked, money earned increases by  $\$10$

*Predicted Values*

If Test Anxiety is 12 and Hours Studied = 3.5, what is the predicted value of Math Achievement?

Math' =  $9.00 + -0.33$  (Test Anxiety) +  $1.45$  (Hours Studied)

Math' =  $9.00 + -0.33$  (12) +  $1.45$  (3.5)

$$= 9.00 + (-3.96) + (5.075)$$

$$= 10.115$$

If Test Anxiety is 5 and Hours Studied is 6.3, what is the predicted value of Math Achievement?

$$\text{Math}' = 9.00 + -0.33 (\text{Test Anxiety}) + 1.45 (\text{Hours Studied})$$

$$\text{Math}' = 9.00 + -0.33 (5) + 1.45 (6.3)$$

$$= 9.00 + (-1.65) + (9.135)$$

$$= 16.485$$

### 3. Predicted Values vs. Expected Change

**Predicted values** obtained from prediction equation:

Prediction Equation

$$\text{Math}' = b_0 + b_1 (\text{Test Anxiety}) + b_2 (\text{Hours Studied})$$

$$\text{Math}' = 9.00 + -0.33 (\text{Test Anxiety}) + 1.45 (\text{Hours Studied})$$

Example:

See examples above

**Expected change** is obtained from the slope coefficient:

$$\text{Expected change in } Y = (b_1) (\text{Change in Test Anxiety})$$

Examples

What change is expected in math achievement for someone who has anxiety level that increases by 3 points (controlling for hours studied)?

$$\text{Expected Change} = -.33 (\text{change in anxiety})$$

$$-.33 (+3)$$

$$= -.99 \text{ point change in math achievement}$$

What change would be expected in math achievement for someone who studies an additional 6 hours (controlling for test anxiety)?

$$\text{Expected Change} = 1.45 (\text{change in Hours Studied})$$

$$1.45 (6)$$

$$= 8.7 \text{ point increase in math achievement}$$

### 4. Obtaining Regression Estimates

See chat notes 11 for continuation of Multiple Regression.