Notes 8a Simple Regression
Supplemental Notes

Figure 1 page 2

(a) $b_0 = 0.00$

(b) $b_1 =$
b1 = rise / run = 1 / 1 = 1.00
Y' = b0 + b1X
Y' = 0.00 + 1.00(X)

(c) Find Y' when X = 5 using the prediction equation
Y' = 0.00 + 1.00(X)
Y' = 0.00 + 1.00(5)
Y' = 0.00 + 5 = 5

(d) Find Y' when X = 2 using the prediction equation
Y' = 0.00 + 1.00(X)
Y' = 0.00 + 1.00(2)
Y' = 0.00 + 2 = 2

(e) Find Y' when X = 12 using the prediction equation
Y' = 0.00 + 1.00(X)
Y' = 0.00 + 1.00(12)
Y' = 0.00 + 12 = 12

(f) Find Y' when X = 0 using the prediction equation
Y' = 0.00 + 1.00(X)
Y' = 0.00 + 1.00(0)
Y' = 0.00 + 0 = 0

(g) What is literal interpretation of b_0?
Predicted value of Y when X = 0.00

(h) What is literal interpretation of b_1?
Predicted change in Y for a 1 unit increase in X, so for each 1 point increase in X, we expect a 1 point increase in Y.

Figure 2 Page 3

(a) $b_0 = 4.00$
(b) $b_1 = ½ = .5$
(c) Find Y' when X = 7 using the prediction equation

$Y' = b0 + b1(X)$
$Y' = 4.00 + 0.5 (X)$
$Y' = 4.00 + 0.5 (7)$
$Y' = 4.00 + 3.5 = 7.5$

(d) Find Y' when X = 2 using the prediction equation
$Y' = 4.00 + 0.5 (X)$
$Y' = 4.00 + 0.5 (2)$
$Y' = 4.00 + 1 = 5.00$

(e) Find Y' when X = 0 using the prediction equation
$Y' = 4.00 + 0.5 (X)$
$Y' = 4.00 + 0.5 (0)$
$Y' = 4.00 + 0.00 = 4.00$

(f) What is literal interpretation of b_0?

b0 = 4.00, so when X = 0.00, then the predicted value for Y is 4.00

Example
x = number of hours studied
y = test grade

Test grade is predicted to be 4.00 when there is no hours studied (= 0.00)

(g) What is literal interpretation of b_1?

The predicted value of Y will increase by 0.5 for each 1 point increase in X

Example
x = number of hours studied
y = test grade

For each additional 1 hour of study, test grades are expected to increase by 0.5.

Figure 3 Page 4

(a) $b_0 = 9.00$

(b) $b_1 = -.3333$

(c) Find Y' when X = 4.5 using the prediction equation
$Y' = b0 + b1(X)$
$Y' = 9.00 + -.3333(4.5)$

Y’ = 9.00 + -1.5 = 7.5

(d) Find Y’ when X = 9 using the prediction equation
Y’ = b0 + b1(X)
Y’ = 9.00 + -.3333(9)
Y’ = 9.00 + -3 = 6

(e) Find Y’ when X = 0 using the prediction equation
Y’ = b0 + b1(X)
Y’ = 9.00 + -.3333(0)
Y’ = 9.00 + 0 = 9

(f) What is literal interpretation of b_0?
The predicted value of Y when X = 0.00

Example
x = number of hours studied
y = test grade

Test grade is predicted to be 9.00 if 0.00 hours are studied.

(g) What is literal interpretation of b_1?
For each 1 point increase in X, Y is predicted to decline -.3333.

Example
x = number of hours studied
y = test grade

For each additional hour studied, test grades will decline by 1/3 of a point.

Section 3
Residuals

Residual: Y – Y’

Example from Figure 3
X = 6
Y’ = 9.00 + -.3333(6)
Y’ = 9.00 + -2 = 7

Example Y = 6
Residual = 6 – 7 = -1

Example Y = 8
Residual = 8 – 7 = 1

Section 4

(a) If X (percentage of students with the grade of A) increases by 10, what is the amount of change expected in student ratings?
b1 = 0.034

Estimate change by multiplying b1 by the amount of change in the IV
Expected change in Y (DV) = b1 * (change in IV)

.034 * 10 = 0.34

Second example showing how to calculate change in Y for a given change in X:

What is the predicted rating when 40% receive grade of A?
Y' = b0 + b1(X)
Y' = 2.47 + 0.034(40) = 3.83

What is the predicted rating when 50% receive grade of A?
Y' = b0 + b1(X)
Y' = 2.47 + 0.034(50) = 4.17

Note the 10 percentage point difference in grade of A above.

The difference in predicted rating will be the amount of change expected for a 10 percentage point increase in students who receive grade of A:

4.17 – 3.83 = 0.34

(b) If X decreases by 5, what is the amount of change expected in student ratings?

.034 * -5 = -0.17

(c) If X = 25, what is the predicted mean student rating?
Y' = b0 + b1(X)
Y' = 2.47 + 0.034(25) = 3.32

(d) If X = 75, what is the predicted mean student rating?
Y' = b0 + b1(X)
Y' = 2.47 + 0.034(75) = 5.02

Section 4

Example 1 Barometric Pressure of Water Boiling Point

(a) What is literal interpretation for b_0 and b_1?

b0 = 155.286 – this is the predicted boiling point of water when barometric pressure equals 0.00

b1 = 1.902 – for each additional increase of 1 point in barometric pressure, the boiling point of water is expected to increase by 1.902 degrees F.

(b) What is the predicted boiling point of water in degrees Fahrenheit if barometric pressure is 21?

Y' = b0 + b1(X)

Boiling Point' = 155.286 + 1.902 (Barometric pressure)

Boiling Point' = 155.286 + 1.902 (21) = 195.286

(c) What is the predicted boiling point of water in degrees Fahrenheit if barometric pressure is 30?

Y' = b0 + b1(X)

Boiling Point' = 155.286 + 1.902 (Barometric pressure)

Boiling Point' = 155.286 + 1.902 (30) = 212.346

(d) If barometric pressure was increased by 9 points, what degrees Fahrenheit change would be expected in the boiling point of water?

b1 = 1.902 (this is the predicted change in the DV for a 1 point increase in the IV)

b1 = 1.902 (9) = 17.118

Example 2 Cotton Yield and Irrigation

(a) What is literal interpretation for b_0 and b_1?

b0 = -24.487 – this represents the predicted yield in lbs. per acre of cotton when there is no irrigation.

b1 = 167.856 -- for each additional 1 foot increase in water irrigation per acre, we expect a 167.856 lbs. increase in cotton yield.

(b) What is the predicted cotton yield in pounds per acre if irrigation is 2.5 feet of water per acre?

Y' = b0 + b1(X)

Cotton yield' = -24.487 + 167.856 (irrigation in feet per acre)

Cotton yield' = -24.487 + 167.856 (2.5) = 395.153

(c) What is the predicted cotton yield in pounds per acre if irrigation is 1.0 foot of water per acre?

Y' = b0 + b1(X)

Cotton yield' = -24.487 + 167.856 (irrigation in feet per acre)

Cotton yield' = -24.487 + 167.856 (1.0) = 143.369

(d) If irrigation was decreased by 1.5 feet of water per acre, what change would be expected in pounds of cotton yield per acre?

b1 = 167.856 *-1.5 = -251.784

Example 3 Car Weight and MPG

(a) What is literal interpretation for b_0 and b_1?

b0 = 49.986 --- this is the predicted MPG for a car that weighs nothing (weight = 0.00)

b1 = -8.778 --- for each additional 1,000 lbs of car weight, MPG is expected to decline by -8.778.

(b) What is the predicted MPG for a car that weighs 2000 lbs?

Y' = b0 + b1(X)

MPG' = 49.986 + -8.778 (car weight)

MPG' = 49.986 + -8.778 (2.0) = 32.43

(c) What is the predicted MPG for a car that weighs 2400 lbs?

Y' = b0 + b1(X)

MPG' = 49.986 + -8.778 (car weight)

MPG' = 49.986 + -8.778 (2.4) = 28.9188

(d) You normally drive alone, but for this upcoming trip two friends will ride with you and their combined weight is 400 lbs. What is the expected change in MPG as a result of your friends riding in your car?

b1 = -8.778 * (change in weight, 400 lbs, but car weight in thousands of lbs so 400/1000 = .4)

b1 = -8.778 * (.4) = -3.5112

Section 6
Overall Model Fit

Finding Critical F-ratios

(1) Example 1 data for Barometric Pressure of Boiling Point of Water: N = 18 and there is one predictor, barometric pressure.

df1 = k = 1

df2 = n – k – 1 = 18 – 1 -1 = 16

critical F (with alpha of .05) = 4.49

critical F (with alpha of .01) = 8.53

(2) Example 2 data for Irrigation and Cotton Yield: N = 14 and there is one predictor, irrigation amount I feet of water per acre.

df1 = k = 1

df2 = n – k – 1 = 14 – 1 -1 = 12

critical F (with alpha of .05) = 4.75

critical F (with alpha of .01) = 9.33

(3) Example 3 data for Car Weight and MPG: N = 10 and there is one predictor, car weight.

df1 = k = 1

df2 = n – k – 1 = 10 – 1 -1 = 8
critical F (with alpha of .05) = 5.32
critical F (with alpha of .01) = 11.26

(4) Example 3 data for Car Weight, engine size in CC, horsepower and MPG: N = 10 and there is three predictors, car weight, engine size, and horsepower.

df1 = k = 3
df2 = n – k – 1 = 10 – 3 -1 = 6
critical F (with alpha of .05) = 4.76
critical F (with alpha of .01) = 9.78